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Analysis and scheduling of practical network coding in OFDMA relay networks

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ABSTRACT

Network coding has become a prominent approach to improve throughput of wireless networks. However, most of work in the literature concentrates mainly on 802.11-like random access networks. New technologies such as OFDMA (orthogonal frequency division multiple access), offer new opportunities for employing network coding. This paper considers how to apply the practical network coding scheme in OFDMA relay networks via cross-layer optimization. Specifically, we aim to explore the following questions: (1) When and how can wireless nodes select relay paths in the presence of network coding? (2) How can an OFDMA relay system assign network resource such as subcarrier and power for all the transmitting nodes? (3) What are the impacts of OFDMA system parameters on the network coding gain? To answer these questions, two efficient coding-aware relay strategies are presented to select forwarding paths with fixed and dynamic power allocation. In order to exploit the network capacity in slow frequency selective fading channels, we formulate optimization frameworks and propose channel-aware coding-aware resource allocation algorithms for an arbitrary traffic pattern. Our studies show that the network coding (i.e. XOR) gain depends on the nodes' powers, traffic patterns etc. Especially, OFDMA relay network with dynamic power possesses both ***coding gain*** and ***power gain***. Extensive simulations are performed to verify our analysis and demonstrate the throughput improvement of our proposals in the presence of XOR coding.

1. Introduction

In the past few years, OFDMA networks such as WiMax and 3G LTE have been brought into commercial deployment. It can support various bandwidth-intensive services such as interactive games, VoIP, peer-to-peer streaming that involve bidirectional traffic. However, the capacity of an OFDMA cell is still limited by communication distance, as many other wireless communication systems. Deploying dense base stations (*BS*) can mitigate the capacity loss caused by large-scale fading, but at the expense of high deployment and management costs. An efficient way is to make use of inexpensive relays to forward traffic between a base station and mobile/portable subscribers (*SS*). When bidirectional flows pass through a relay station (*RS*), the needed resource can be potentially saved if network coding is introduced.

The practical XOR coding scheme [2] demonstrates throughput enhancement in 802.11 networks, which uses a single common frequency. However, very limited research concentrates on applying network coding in "*OFDMA relay networks*", where the fractional frequency band, namely *subcarrier*, is the allocable resource. To the best of our knowledge, Zhang et al. [11] initially study the performance of XOR operation in the base station to encode packets for subscribers in the OFDMA cell. Practical network coding can reduce traffic load in a relay station so that a number of orthogonal "subcarriers" can be saved

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for other applications. An illustration of coding structure is shown in Fig. 1, where the relay node can encode bidirectional flows using one subcarrier instead of two subcarriers. Although the OFDMA system shares the same coding structure with the 802.11 ad hoc networks, they are fundamentally different in the coding operation, the coding gain and the scheduling methods. Previous studies (e.g. [2]) identify that this coding structure has a XOR coding gain of 4/3 in 802.11 wireless networks. While in an OFDMA *relay* system, we analytically show that the coding gain depends on the traffic pattern, nodes' powers and channel gains. We also have an interesting observation on the effects of network coding when a node's power can be dynamically allocated. In the relay station, XOR operation not only reduces the traffic loads, but also shifts the saved power on other transmitting subcarriers, resulting in an additional ***power gain***.

Unlike the distributed random access scheme of IEEE 802.11, the orthogonal frequency band of an OFDMA relay network is allocated by the base station. Intuitively, if a *RS* transmits a set of bidirectional traffic that can be encoded together, the *BS* assigns less subcarriers to the *RS*. Thus, the network can support a larger traffic matrix with coding-aware relaying. A fundamental question is how to allocate network resource (e.g. subcarrier and power) to optimize the network throughput for an arbitrary traffic pattern when network coding is enabled. This problem, though very important, has not been studied in the literature. Authors in [10] proposed an adaptive resource allocation scheme to optimize OFDMA downlink throughput with fairness constraints. Based on their work, we propose channel-aware and coding-aware scheduling algorithms to scale up the OFDMA system throughput.

Another important question is how to select routing in the presence of multiple relays. We illustrate this problem in Fig. 2 with bidirectional flows between the *BS* and the *SS*. Without network coding, the uplink and the downlink may choose different relay paths based on their individual benefits. For example, the *BS* prefers RS_1 while the *SS* is inclined to select RS_2. When network coding is considered jointly with route discovery, the uplink and the downlink flows may traverse the same path, e.g. $BS - RS_2 - SS$, that can bring potential XOR-coding gain and improve resource usage.

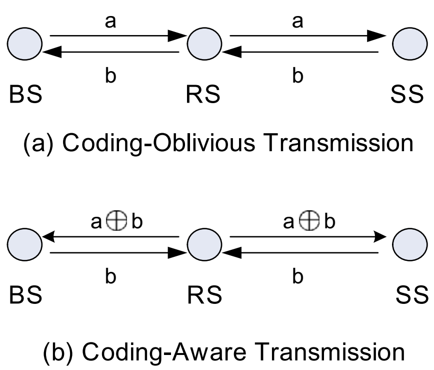

Fig. 1. XOR for OFDMA relay network in reducing number of subcarriers.

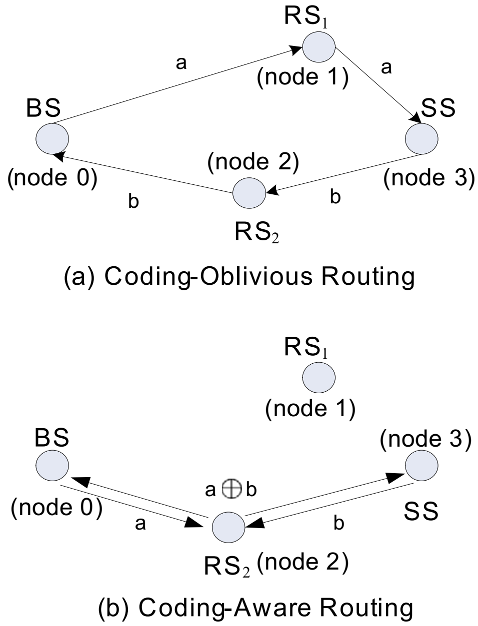

Fig. 2. XOR route selection.

The major contributions of our paper are:

- Two coding-aware routing strategies are presented to facilitate XOR-coding in OFDMA relay networks with fixed and dynamic power allocations.
- We formulate optimization frameworks to perform resource allocation (e.g. subcarrier, power) for OFDMA relay networks in the presence of network coding.
- We propose channel-aware coding-aware scheduling algorithms to maximize system throughput for an arbitrary traffic pattern.
- We investigate the impact of transmission power, channel gain, traffic pattern on the network coding gain, and find (a) the throughput gain of XOR-coding depends on the transmission powers of wireless nodes; (b) dynamic power allocation brings both *coding gain* and *power gain*.

The rest of this paper is organized as follows. In Section 2, we describe the basic characteristics of an OFDMA relay system. Section 3 presents the XOR-coding scheme and the coding-aware routing strategies. In Sections 4 and 5, coding-aware channel-aware scheduling algorithms are proposed with fixed and dynamic power allocation, respectively. We compare the end-to-end throughput of the coding-aware algorithms with those of the coding-oblivious algorithms in Section 6. Literature survey is presented in Section 7 and Section 8 concludes.

2. System description

In OFDMA mechanism, the frequency band is divided into a set of orthogonal narrow band subcarriers. Although OFDMA has many desirable characteristics, its performance is still limited by transmission distance, as many other wireless communication systems. To deal with this issue, the OFDMA mechanism is designed to enable relaying of traffic over multiple hops using the mesh mode. The mesh mode provides both high throughput and an increased coverage range with the help of inexpensive relay stations.

Fig. 3 illustrates a cellular OFDMA relay network that consists of a base station (BS), M relay station (RS) as well as N subscribers (SS). The sets of RS and SS are denoted to be $\mathscr{M}$ and $\mathscr{N}$. The relay stations are stationary and serve as mesh routers to forward packets between BS and SS. The subscribers are located at diverse geographic locations and connect to the BS either directly (one-hop from the BS), or indirectly via a RS (two-hop away from the BS). The BS, labeled as node 0, has a set of directly connected downlink nodes $\mathscr{D}_0$ and a set of directly connected uplink nodes $\mathscr{U}_0$. Denote $\mathscr{V}$ to be the set of all wireless nodes in the network, $\mathscr{V} = \mathscr{M} \cup \mathscr{N} \cup \{0\}$. Note that a node can be the BS, the RS or the SS. For each relay station m, we denote $\mathscr{D}_m$ and $\mathscr{U}_m$ to be the sets of direct downlink and uplink nodes, and denote $\mathscr{V}_m$ to be the set of nodes directly connected to the mth RS, $\mathscr{V}_m = \mathscr{U}_m \cup \mathscr{D}_m$. Here, the uplink of a RS to the BS is exactly the downlink of that RS to the BS if we observe from the perspective of the RS. When the relay paths are determined, the end-to-end flows between the BS and the SSs can be represented by a set of *directed* links, denoted as $\mathscr{L}$. Then, the topology of the OFDMA relay network can be modeled as a *directed* graph $\mathscr{G} = \{\mathscr{V}, \mathscr{L}\}$. In order to avoid repeated counting of the same links, the set $\mathscr{D}_0$ includes the relay stations while the set $\mathscr{U}_0$ excludes the relay stations. The set $\mathscr{D}_0$ has D_0 nodes and the set $\mathscr{U}_0$ has U_0 nodes. Similarly, the sets $\mathscr{D}_m, (m \in \mathscr{M})$ contain the base station 0, yet the sets $\mathscr{U}_m, (m \in \mathscr{M})$ do not contain the BS. In other words, the set $\mathscr{D}_m$ has D_m nodes that consist of the BS and $D_m - 1$ subscribers, whereas the set $\mathscr{U}_m$ is composed of only U_m subscribers. By breaking up the wireless links for the BS and the RSs, the OFDMA relay network is decomposed into a class of single-hop infrastructure access networks with shared frequency band. Every direct uplink subscriber of the BS or the RSs has a link flow. To evaluate the network performance, we introduce the concept "*traffic pattern*" which is the direction of the flow rate vector or the ratio of the flow rates. When the relay paths are selected, the end-to-end traffic pattern can be mapped into the traffic pattern of all links due to the property of *flow conservation*. For a given relay/coding scheme, the *throughput* of a flow is defined as the rate in *bit-per-second* or *bps*. Let $\gamma = \{\gamma_{ij} | i,j \in \mathscr{V}\}$ be an arbitrary traffic pattern, our objective is to maximize the aggregate throughput of end-to-end flows constrained by the traffic pattern γ. We define an alternative performance metric, the "*scaling factor*" (denoted as θ_{ij}), that is computed by the throughput of a flow (e.g. $\overrightarrow{ij}$) over the corresponding traffic ratio (e.g. γ_{ij}). The scaling factor of a traffic pattern, θ, is defined as the minimum value of link scaling factors $\theta = \min\{\theta_{ij} | \overrightarrow{ij} \in \mathscr{L}\}$. Hence, the throughput maximization problem for the traffic pattern γ is equivalent to finding the maximum scaling factor θ among all links of the OFDMA relay network.

The total bandwidth of the OFDMA relay network is equally divided into a set of K non-overlapping orthogonal subcarriers denoted by $\mathscr{K}$, each of which has a bandwidth of W Hz. These subcarriers are shared by all links at the BS, RSs and SSs. We simply assume that the BS gathers the channel quality information of all the links and plays the role of assigning subcarriers to all the transmitters in a time slot. Here, *time slot* defines the minimum time resource that can be allocated by an OFDMA relay system to a given link. Although our assumption might not hold when the BS cannot obtain the channel information of links two-hop away, the subcarrier assignment can still be carried out similar to [12] via two levels. In the first level, the BS assigns subcarriers to the one-hop nodes based on the average channel qualities and the average traffic loads. In the second level, the relay stations schedule the transmission of mobile subscribers opportunistically by exploiting multiuser diversity. The transmission power of the BS is denoted to be P_0. The transmission powers of the mth RS $(m \in \mathscr{M})$ and the ith SS $(i \in \mathscr{N})$ are denoted to be P_m and P_i, respectively. The base station, the relay stations and the mobile subscribers are constrained by their maximum transmission powers P_0, P_m $(m \in \mathscr{M})$ and P_i $(i \in \mathscr{N})$. Note that the transmission powers of RSs can be different. Likewise, P_i can also be different for $i \in \mathscr{N}$. Let h_{ij}^k be the channel gain of link $\overrightarrow{ij}$ at subcarrier k and p_{ij}^k be the corresponding transmission power. When the M-ary quadrature

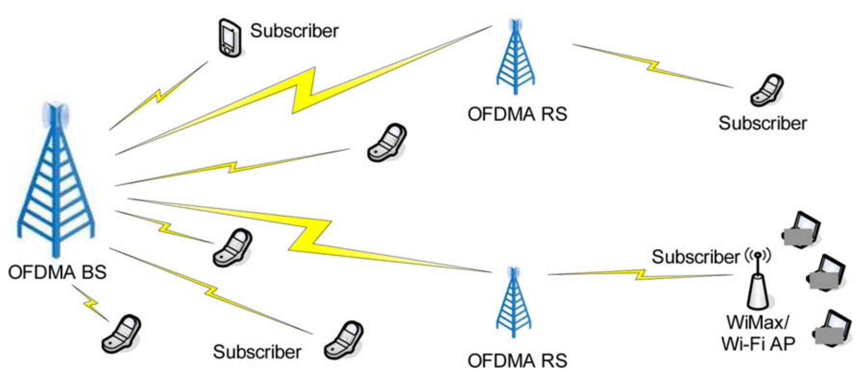

Fig. 3. Topology of an OFDMA relay network.

amplitude modulation (MQAM) is adopted, c_{ij}^k, the channel rate of link $\overrightarrow{ij}$ at subcarrier k for a given BER, can be expressed as

$$c_{ij}^k = W\log_2\left(1 + \beta\frac{p_{ij}^k h_{ij}^k}{\sigma^2}\right), \quad (1)$$

where σ^2 is the noise level and β is the gap to the link capacity. Here, we denote a new variable, the channel quality g_{ij}^k, to be $\beta h_{ij}^k/\sigma^2$ for brevity. We summarize the physical meanings of the variables in Table 1 for clarity.

3. Network coding, routing and a unified model

In this section, we first present the XOR coding scheme to encode flows at OFDMA relay stations. Two coding-aware traffic-aware routing schemes are proposed to enable XOR-coding for fixed and dynamic power allocations, respectively.

3.1. Encoding flows in the relay stations

We first define the terminologists that will be used throughout this paper. For the XOR coding, a *coding node* is the node which encodes packets for several flows, e.g. *RS* in Fig. 1. *Encodable flows* are flows that transmit via a coding node and their packets have the opportunity to be encoded (e.g. flows $BS \to RS \to SS$ and $SS \to RS \to BS$ in Fig. 1a). *Encoded link flow* defines the new link flow at the coding node after XOR operation. A *coding structure* includes one coding node (*RS*) as well as the one-hop predecessor nodes and the one-hop successor nodes of the associated coding flows [6]. *Load* means the bits per-second transmitted over a source-destination pair or a link. *Average traffic load* is the traffic load averaged over time. *Coding gain* denotes the ratio of the coding-aware scaling factor, to the coding-oblivious scaling factor, in which the gain is originated from the reduced loads at the coding nodes. *Forwarding path* defines a directed path that the *BS* or a *SS* delivers the traffic load via a relay station.

Table 1
Notations of variables in the OFDMA relay networks.

Notations	Physical meanings
$\mathcal{N}$	the set of subscribers
$\mathcal{M}$	the set of relay stations
$\mathcal{L}$	the set of links
$\mathcal{V}$	the set of nodes including the *BS*, *RS*s and *SS*s
$\mathcal{G}$	the directed graph for the considered topology
$\mathcal{D}_0$	the set of downlink nodes directly connected to the *BS*
$\mathcal{U}_0$	the set of uplink nodes directly connected to the *BS*
$\mathcal{D}_m$	the set of downlink nodes connected to the *m*th *RS*
$\mathcal{U}_m$	the set of uplink nodes connected to the *m*th *RS*
$\mathcal{V}_m$	the set of nodes connected to the *m*th RS, $\mathcal{V}_m = \mathcal{U}_m \cup \mathcal{D}_m$
$\mathcal{K}$	the set of subcarriers
W	the bandwidth of a subcarrier
γ	the traffic pattern (ratios of flow rates)
γ_{ij}	the traffic ratio of link $\overrightarrow{ij}$ in the traffic pattern γ
θ_{ij}	the scaling factor of link $\overrightarrow{ij}$
θ	the scaling factor of the traffic pattern $\gamma, \theta = \min\{\theta_{ij} \mid \overrightarrow{ij} \in \mathcal{L}\}$
r_{ij}	the throughput of link $\overrightarrow{ij}$, provided the allocated resource
P_0	the transmission power of the base station
P_m	the transmission power of the *m*th *RS* for $m \in \mathcal{M}$
P_i	the transmission power of the *i*th *SS* for $i \in \mathcal{N}$
h_{ij}^k	the channel gain of link $\overrightarrow{ij}$ at the *k*th subcarrier
g_{ij}^k	the channel quality of link $\overrightarrow{ij}$ at the *k*th subcarrier, $g_{ij}^k = \frac{\beta h_{ij}^k}{\sigma^2}$
p_{ij}^k	the transmission power of link $\overrightarrow{ij}$ at the *k*th subcarrier
c_{ij}^k	the channel rate of link $\overrightarrow{ij}$ at the *k*th subcarrier
Ω_{ij}	the set of subcarriers assigned to the link $\overrightarrow{ij} \in \mathcal{L}$
Ω_i	the set of subcarriers assigned to the node $i \in \mathcal{V}$

Authors in [2] proposed the COPE scheme to XOR packets opportunistically in 802.11 networks. However, the physical characteristics of OFDMA system are quite different, which leads to the remarkable distinctions in applying XOR-coding. In contrast to 802.11, OFDMA adapts the modulation and coding scheme (MCS) according to the quality of radio channel, and thus the bit rate and robustness of data transmission. Hence, when coding bidirectional flows together, the relay station should take the channel gains of receivers into consideration. In general, XOR coding has two types of scenarios: *bidirectional data transfer* and *opportunistic listening*. However, we only consider the bidirectional data transfer scenario for the following reasons:

- The mobility of subscribers causes the overhearing range to be time-varying. Therefore, the subscribers and the relays need to frequently exchange the channel gains with each other to build the coding structures for opportunistic listening.
- Different relay selections usually have different overhearing ranges, which requires an exhaustive search of optimal coding opportunity and routing.
- Frequency selective fading usually prevents one subscriber from fully overhearing the other subscriber over the corresponding subcarriers.

To explain the first and the third difficulties of opportunistic listening, we adopt the following case study shown in Fig. 4. There are two flows, $BS \to RS \to SS_1$ and $SS_2 \to RS \to BS$, that traverse the common relay station. If SS_1 and SS_2 are close enough, SS_2 is able to overhear the transmission of SS_1. Thus, the coding node, *RS*, can encode the packets of these two flows. For instance, packets a and b are mixed in *RS* and the new packet $a \oplus b$ is broadcasted to BS, SS_1 and SS_2. However, opportunistic listening is sometimes not practical even in this simple network scenario. First, we consider a static wireless channel. Assume that the distance between SS_1 and *RS* is less than that between SS_1 and SS_2. The successful overhearing range of SS_1 is shown in Fig. 4a if SS_1 transmits to *RS* using the data rate according to Eq. (1). Thus, SS_2 cannot decode the overheard packets of SS_1 because SS_2 is outside of the successful overhearing range. To enforce network coding, SS_1 must transmit using a lower rate modulation so as to increase the overhearing range, as shown in Fig. 4b. One can see that opportunistic listening greatly increases the complexity of scheduling, while the benefit is usually unknown. Second, the channel gains are time-varying in OFDMA systems. In some subcarriers, SS_1 can successfully overhear SS_2, while fails in some other subcarriers. Hence, SS_2 might not receive a correct packet b and cannot decode the packet $a \oplus b$. Due to these challenges, opportunistic listening is not considered in this work.

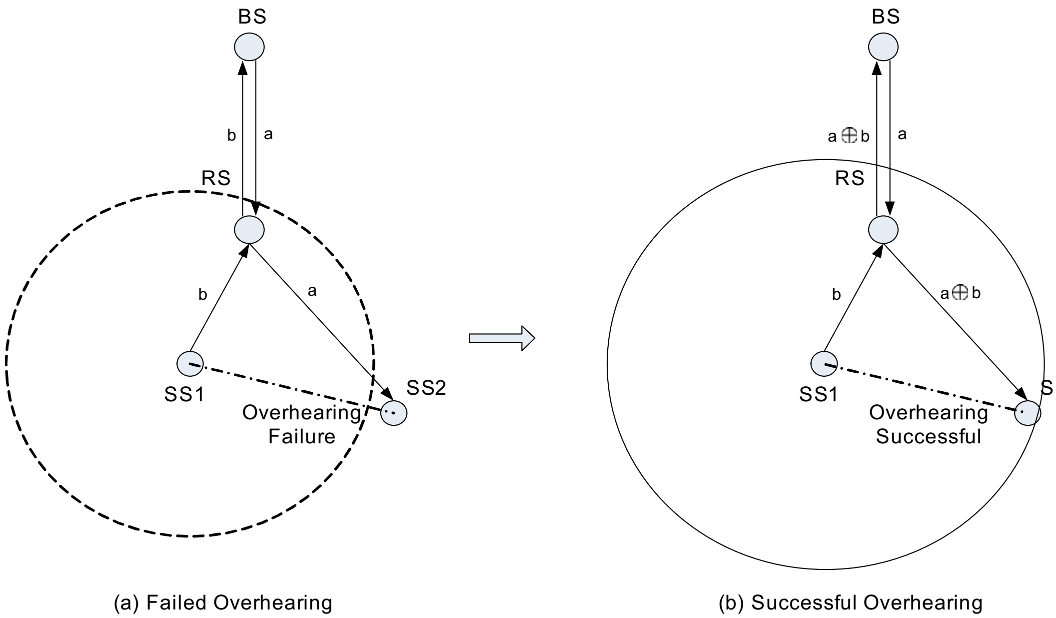

Fig. 4. The challenges of utilizing opportunistic listening.

Next, we present a simple scheme on how to encode bidirectional flows using XOR operation. A relay station combines the same amount of encodable flows into a coded one from the subscriber and the base station. This coded flow is then broadcasted using the data rate that can be decoded by the receiver with the worse channel state. Here, we show how to implement XOR-coding in the relay station m. Assume the end-to-end traffic $\overrightarrow{0i}$ has a larger load than that of $\overrightarrow{i0}$, i.e. $\gamma_{0i} > \gamma_{i0}$. If they traverse the same relay station m, the link traffic from the relay to the BS is $\gamma^{i}_{m0} = \gamma_{i0}$ and the traffic from the relay to the SS is $\gamma_{mi} = \gamma_{0i}$ due to flow conservation constraints. The mth RS mixes γ^{i}_{m0} and γ_{mi} together to create a coded link flow $\gamma^{c}_{mi} = \gamma_{i0}$ and a remaining link flow $\gamma^{r}_{mi} = \gamma_{0i} - \gamma_{i0}$. Similarly, we create a set of coded links γ^{c}_{mi} and a set of remaining links $\gamma^{r}_{mi}, (i \in \mathscr{D}_m)$ for multiple encodable link flows at the relay m. Denote $\mathscr{D}^{c}_{m}$ to be the set of nodes whose data rates correspond to those of encoded flows. Denote $\mathscr{D}^{r}_{m}$ to be the set of nodes that still have remaining flows after the XOR-coding operation. We replace the original link flows (before XOR coding) by the coded flows and the remaining flows at each relay station. Note that the empty remaining flows are removed from the set $\mathscr{D}^{r}_{m}$. The newly defined variables are summarized in Table 2 and a systematic XOR-coding algorithm is shown in Fig. 5.

Table 2
Notations of variables in the XOR-coding.

Notations	Physical meanings
$\mathscr{D}^{c}_{m}$	the set of receivers of encoded link flows after XOR-coding at the mth RS
$\mathscr{D}^{r}_{m}$	the set of receivers of remaining flows after XOR-coding at the mth RS
γ^{c}_{mi}	the traffic ratio of the coded link flow $\overrightarrow{mi}$ at the mth RS
γ^{r}_{mi}	the traffic ratio of the remaining link flow $\overrightarrow{mi}$ at the mth RS
r^{c}_{mi}	the throughput of the coded link $\overrightarrow{mi}$
r^{r}_{mi}	the throughput of the remaining link $\overrightarrow{mi}$
$g^{c,k}_{mi}$	the channel quality of the coded link $\overrightarrow{mi}$ at subcarrier k
$g^{r,k}_{mi}$	the channel quality of the remaining link $\overrightarrow{mi}$ at subcarrier k

We further introduce a simple example to depict how the XOR-coding scheme works.

Example 1. Consider a relay structure with two subscribers and four bidirectional flows in Figs. 6 and 7. The flow $BS \rightarrow SS_1$ can be encoded with the flow $SS_1 \rightarrow BS$. Likewise, $BS \rightarrow SS_2$ can be mixed with $SS_2 \rightarrow BS$. The end-to-end flow pattern $\{BS \rightarrow SS_1, BS \rightarrow SS_2, SS_1 \rightarrow BS, SS_2 \rightarrow BS\}$ is $\{0.1, 0.2, 0.4, 0.3\}$. The goal of resource allocation is to find a maximum factor to scale up this traffic pattern, provided the power and subcarrier constraints. Due to the flow conservation property, we obtain a set of traffic for all links.

After XOR-coding, we construct two traffic vectors in the relay station described by Table 3. The remaining links $\gamma^{1,r}_{m0}$ and $\gamma^{2,r}_{m0}$ have traffic ratios of 0.4 and 0.3. The RS combines the links $RS \rightarrow SS_1$ and $RS \rightarrow BS$ of bidirectional flows $BS \leftrightarrow SS_1$, thus creating a coded flow with a traffic demand 0.1. In order to ensure the successful decoding, the XOR-coded flow is transmitted based on the worse channel gain of both receivers. Since the traffic loads of uplink and downlink are not symmetric, the remaining link is updated as $\gamma^{r}_{m0} = 0.4 + 0.3 - 0.1 = 0.6$. Similarly, when the bidirectional flows $BS \leftrightarrow SS_2$ are encoded by the RS, another XOR-coded flow is generated with a ratio of 0.2 and the ratio of the remaining link flow to the BS is $\gamma^{r}_{m0} = 0.6 - 0.2 = 0.4$. When a frequency selective multicarrier system is considered, the channel gain of the coded flow in a subcarrier is the smaller channel gain of encodable flows at that slot.

XOR coding operation

Initialization

1: Let $\mathcal{D}_m^c = \varnothing$ and $\mathcal{D}_m^r = \mathcal{D}_m$;
2: link $\overrightarrow{mi}$ is rewritten as $\overrightarrow{mi}^r$, for $i \in \mathcal{D}_m$ and $m \in \mathcal{M}$;
3: Let the ratios of remaining link flows be: $\gamma_{mi}^r = \gamma_{mi}$
    for all $i \in \mathcal{D}_m$;

Encoding Flows

4: **If** the end-to-end flows $\overrightarrow{0i}$ and $\overrightarrow{i0}$ traverse the m^{th} RS
5:    add node i to the set $\mathcal{D}_m^c$: $\mathcal{D}_m^c = \mathcal{D}_m^c \cup \{i\}$;
6:    $\mathcal{L} = \mathcal{L} \cup \{\overrightarrow{mi}^c\}$; // add the coded link
7:    set the ratio of the coded link flow:
    $\gamma_{mi}^c = \min\{\gamma_{0i}, \gamma_{i0}\}$; //$\gamma_{0i}$ and γ_{i0} are end-to-end ratios
8:    set the channel quality of the coded link flow:
    $g_{mi}^{c,k} = \min\{g_{m0}^k, g_{mi}^k\}$;
9:    update the ratios of the remaining link flows:
10:      **If** $\gamma_{0i} \geq \gamma_{i0}$
        //downlink traffic is larger than uplink traffic
11:        $\gamma_{mi}^r = \gamma_{mi}^r - \gamma_{mi}^c$;
        //update the ratio of the remaining link $\overrightarrow{mi}$
12:        $\gamma_{m0}^r = \gamma_{m0}^r - \gamma_{mi}^c$;
        //update the ratio of the remaining link $\overrightarrow{m0}$
13:        **If** $\gamma_{m0}^r = 0$
14:          $\mathcal{D}_m^r = \mathcal{D}_m^r \backslash \{0\}$; // remove BS from the set $\mathcal{D}_m$
15:          $\mathcal{L} = \mathcal{L} \backslash \{\overrightarrow{m0}^r\}$; // remove remaining link $\overrightarrow{m0}^r$
16:        **End if**
17:      **Else if** $\gamma_{0i} < \gamma_{i0}$
        //downlink traffic is smaller than uplink traffic
18:        $\gamma_{mi}^r = 0$;
        //update the ratio of the remaining link $\overrightarrow{mi}$
19:        $\gamma_{m0}^r = \gamma_{m0}^r - \gamma_{mi}^c$;
        //update the ratio of the remaining link $\overrightarrow{m0}$
20:        $\mathcal{D}_m^r = \mathcal{D}_m^r \backslash \{i\}$;
        // remove the i^{th} node from the set $\mathcal{D}_m$
21:        $\mathcal{L} = \mathcal{L} \backslash \{\overrightarrow{mi}^r\}$;
        // remove the remaining link $\overrightarrow{mi}^r$
22:      **End if**
23:      $\gamma_{0i} = 0$ and $\gamma_{i0} = 0$; // clear end-to-end ratios temporarily.
24: **End if**
25: repeat 4∼24 for each pair of encodable end-to-end flows;

Fig. 5. XOR-coding operation in the mth relay station.

3.2. Coding-aware routing with fixed power allocation

In this subsection, we present a coding-aware routing scheme for the OFDMA relay network where the transmission power of each node is evenly distributed on all subcarriers, that is, p_{ij}^k is replaced by $\frac{P_i}{K}$ at the ith transmitter. The scheduling with even power distribution approximates some practical MAC protocols such as TDMA-OFDM [13] that a node utilizes the whole spectrum when scheduled. Under this situation, the total transmission power is evenly distributed across all subcarriers. Although the TDMA-OFDM MAC is different from the fixed-power subcarrier assignment scheme, they share the same property in the relay networks that the total power is evenly distributed over all the subcarriers at each transmitting node. In this subsection, we investigate the routing strategy for both static channels and slow frequency selective fading channels.

In order to highlight the coding-aware routing and scheduling, we first assume flat fading channels for all nodes so that the whole frequency components of the signal experience the same magnitudes of fading.[1] Hence, we remove the superscript k in the variables c_{ij}^k and g_{ij}^k. Let x_{ij} be the number of subcarriers used by link $\overrightarrow{ij}$, the throughput r_{ij}, is expressed as

[1] Later on in this section, we extend the coding-aware scheduling to a slow frequency selective fading environment.

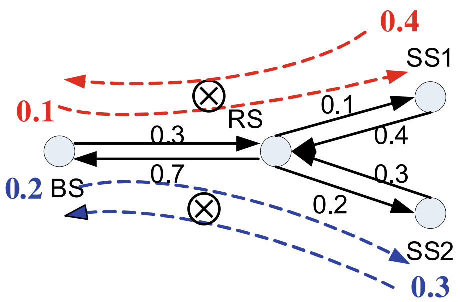

Fig. 6. Before XOR-coding.

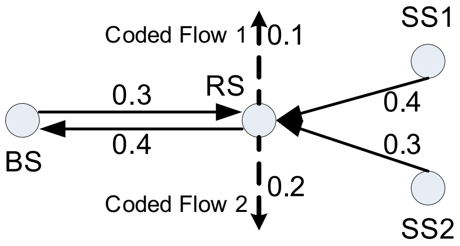

Fig. 7. After XOR-coding.

Table 3
Traffic pattern and channel gains after XOR-coding.

Coded ratio	Coded channel quality
$\gamma^c_{m1} = 0.1$	$g^{c,k}_{m1} = \min\{g^k_{m0}, g^k_{m1}\}$
$\gamma^c_{m2} = 0.2$	$g^{c,k}_{m2} = \min\{g^k_{m0}, g^k_{m2}\}$
Remaining load	Remaining channel gain
$\gamma^r_{m1} = 0, \gamma^r_{m2} = 0$	$\emptyset$
$\gamma^r_{m0} = 0.4$	g^k_{m0}

$$r_{ij} = x_{ij}c_{ij} = x_{ij}W\log_2\left(1 + \frac{P_i g_{ij}}{K}\right). \tag{2}$$

Then, the needed number of subcarriers is $x_{ij} = r_{ij}/\left(W\log_2\left(1 + \frac{P_i g_{ij}}{K}\right)\right)$. A source-destination pair $\overrightarrow{ij}$ may select a relay station RS_m to forward its traffic when the relay mode can reduce the needed subcarriers compared with direct communication. That is, the relay mode is beneficial when the following inequality holds

$$x_{ij} > x_{im} + x_{mj}, \tag{3}$$

where either i or j is the *BS*.

Next, we consider the realistic OFDMA channels with frequency selective fading. The number of needed subcarriers for link $\overrightarrow{ij}$ in a slot can be expressed as:

$$x_{ij} = \frac{r_{ij} \cdot K}{\sum_{k=1}^{K} c^k_{ij}} = \frac{r_{ij} \cdot K}{\sum_{k=1}^{K} W\log_2\left(1 + \frac{P_i g^k_{ij}}{K}\right)}. \tag{4}$$

Generally, the channel gains between the *BS* and the *RS*s are static across the subcarriers and time. The time-varying channel fading is limited to the links between the *RS*s and the *SS*s, or between the *BS* and the *SS*s. We further average the needed number of subcarriers over time (e.g. mostly recent 2s) and obtain the median number of needed subcarriers over the subcarriers and time. When the median numbers of needed subcarriers, $\bar{x}_{im}, \bar{x}_{mj}$ and $\bar{x}_{ij}$ satisfy Eq. (3), the relaying can reduce the needed resource in the transmission. For the practical considerations, the transmitting nodes might not be able to compute the average number of needed subcarriers in the whole routing update interval (e.g. 50 s). Instead, the transmitting nodes simply calculate the median number of needed subcarriers based on the channel gains in the past several seconds (e.g. 2 s).

A naive implementation of the principle in Eq. (3) either misses coding opportunities or leads to the inefficient relay selection. To fully utilize the advantage of network coding, the *BS* and the *SS*s need to exploit all possible paths. But the relay stations available for a subscriber are usually limited in a cell. Only a couple of them can help to improve the bandwidth efficiency of a subscriber. This property greatly reduces the complexity of selecting relay paths. Each subscriber has two or three candidate relay stations that are judged by their channel gains. Let us take Fig. 2(a) as an example that has two relay stations. A downlink flow has three routing strategies: using RS_1, using RS_2 and direct transmission. If there exist bidirectional flows, they have nine routing strategies. To resolve the conflicts of relaying services among multiple links, a relay station does not allow or process simultaneous requests. This is also true in realistic network scenarios because each relay station is usually associated with limited number of subscribers. An efficient coding-aware routing is determined through the following steps:

1. A *SS* broadcasts a REQUEST message on the control channel when the average channel gain is below a certain threshold. The REQUEST message contains its average scaled traffic load.
2. A *RS* calculates the number of needed subcarriers according to Eq. (2) or (4), and send it back to the *SS*. Then, the *RS* returns an ACK message to the *SS* that contains the number of aggregate needed subcarriers along this path. If the requesting flow has an encodable flow in the reverse direction, the *BS* computes the needed subcarriers from the *BS* to the *RS* and piggybacks this information to the *RS*. The *RS* computes the needed subcarriers for the coded flow and the remaining flow. The information of aggregated needed subcarriers is send to both the *BS* and the *SS*.
3. When the source node receives several ACK messages, it selects a path that uses the smallest number of subcarriers to transmit the given traffic load.
4. The routing strategy of every *SS* is updated after a random time interval in order to adapt to the traffic dynamics. The average update interval is set to 50 s to suppress the overheads.
5. If the *BS* requests relay service, it also follows the steps from 1 to 4.

3.3. Coding-aware routing with dynamic power allocation

The *dynamic power allocation* means that the total power is dynamically allocated to the assigned subcarriers for each transmitting node. The routing strategy with dynamic power allocation is similar to that with fixed power allocation. A source node requests relay service from the relay stations and decides a forwarding path based on the feedbacks. The major difference is that the throughput of a relay station with dynamic power allocation is not proportional to the number of assigned subcarriers. Hence, path selection with dynamic power control depends on not only the existing flows in the relay stations that can be encoded, but also those that cannot. In order to clearly deliver the problem and the challenge, we focus on the OFDMA relay network with static channels first, and extend to the fading channels subsequently. Denote p_{ij} to be the allocated power for the link $\overrightarrow{ij}$. The throughput of a link $\overrightarrow{ij}$ is characterized by

$$r_{ij} = f(x_{ij}) = x_{ij} W \log_2 \left(1 + \frac{p_{ij} g_{ij}}{x_{ij}}\right), \quad (5)$$

where x_{ij} is the number of assigned subcarriers. Assume that the number of assigned subcarriers can be fractional (i.e. x_{ij} is continuous), we have the following result:

Theorem 1. *The throughput of a link is a strictly increasing, concave function in terms of the allocated subcarriers.*

Proof. Since x_{ij} is regarded as a continuous variable within the range $(0, K]$, and r_{ij} is twice-differentiable, we can examine the convexity by taking the second-order differential of r_{ij} over x_{ij}. The first-order derivative is expressed as

$$\frac{\partial r_{ij}}{\partial x_{ij}} = W \log_2 \left(1 + \frac{p_{ij} g_{ij}}{x_{ij}}\right) - \frac{W p_{ij} g_{ij}}{\log 2 \cdot (x_{ij} + p_{ij} g_{ij})}$$

And the second-order derivative is

$$\frac{\partial^2 r_{ij}}{\partial x_{ij}^2} = \frac{W p_{ij} g_{ij}}{\log 2} \left(\frac{1}{(x_{ij} + p_{ij} g_{ij})^2} - \frac{1}{x_{ij}^2 + p_{ij} g_{ij} x_{ij}}\right) < 0.$$

Hence, the data rate r_{ij} is a concave function of the allocated subcarriers x_{ij} in the range $(0, \infty)$ and $\frac{\partial r_{ij}}{\partial x_{ij}}$ is strictly decreasing. When x_{ij} approaches infinity,

$$\lim_{x_{ij} \to \infty} \frac{\partial r_{ij}}{\partial x_{ij}} := \frac{W p_{ij} g_{ij}}{\log 2 \cdot x_{ij}} - \frac{W p_{ij} g_{ij}}{\log 2 \cdot (x_{ij} + p_{ij} g_{ij})} = 0.$$

Thus, the data rate r_{ij} is a strictly increasing and concave function of the allocated subcarriers x_{ij}. □

According to Theorem 1, Eq. (4) is a concave and strictly increasing function of the number of subcarriers, x_{ij}. The physical meaning is that the throughput of a node increases along with the number of allocated subcarriers, but the contribution of more subcarriers on throughput is not linear, but diminishing. For instance, if a link is allocated with 20 subcarriers, its throughput is less than twice of the throughput when it is assigned with 10 subcarriers. Intuitively, a node is inclined to select a *RS* that has both good channel gains and low existing traffic loads. Later on, we illustrate the relationship between throughput and the number of subcarriers in Fig. 9.

Next, we will show how to compute the needed subcarriers to support the traffic demand of a link $\overrightarrow{ij}, r_{ij}$, for all $\overrightarrow{ij} \in \mathscr{L}$. Since x_{ij} has no explicit solution in Eq. (5), we have to compute x_{ij} using numerical approaches. For an uplink subscriber, x_{ij} can be obtained in Eq. (5) through a binary search. The basic idea is to start from the number of total subcarriers $x_{ij} = K$, and calculate the right side of Eq. (5). If it is greater than r_{ij}, we decrease x_{ij} by half and compute the throughput. If the right side is less than r_{ij}, the number of subcarriers is set to the median value of the current number and the last number. We repeat the searching process until a fixed-point solution is found. When it comes to the *BS* or the *RS*s, they support multiple downlinks. Let us take the mth relay station as an example. Given the traffic demand vector $\{r_{mi}, \forall i \in \mathscr{D}_m\}$, the number of needed subcarriers for the link throughput r_{mi} can be expressed as:

$$x_{mi} = \frac{r_{mi}}{W \log_2 (1 + P_m g_{mi} / x_m)}, \quad (6)$$

where x_m is the number of needed subcarriers at the mth *RS*. Sum up x_{mi} for all $i \in \mathscr{D}_m$, there has

$$\sum_{i \in \mathscr{D}_m} x_{mi} = x_m = \sum_{i \in \mathscr{D}_m} \frac{r_{mi}}{W \log_2 (1 + P_m g_{mi} / x_m)}. \quad (7)$$

If XOR-coding is adopted in the *RS*, we have the following alternative equation:

$$x_m = \sum_{i \in \mathscr{D}_m^c} \frac{r_{mi}^c}{W \log_2 (1 + P_m g_{mi}^c / x_m)} + \sum_{i \in \mathscr{D}_m^r} \frac{r_{mi}^r}{W \log_2 (1 + P_m g_{mi}^r / x_m)}. \quad (8)$$

The above implicit functions can be easily solved through the binary search method mentioned above. Once x_m is obtained, the required subcarrier numbers to relay the link flows are solved subsequently. By comparing the total subcarriers of the nodes in different **paths**, an end-to-end flow or the bidirectional flows can pick favorable path(s) to relay the traffic load(s). There is one major difference between the fixed power relaying and dynamic power relaying. In the former, the needed subcarriers of other links are not affected by the current routing decision. Hence, the requesting node(s) only need to compare the subcarrier demands of interrelated links at different paths. While in the latter, the current relay decision also affects the subcarrier demands of other links in their common nodes. To select a routing path, the requesting node needs to compare the needed subcarriers of wireless nodes (instead of links) at different paths. In brief, the relay selection with dynamic power is not as straightforward as that with fixed power.

The relay selection is very difficult in frequency selective fading channels, even when the transmission power is assumed to be evenly distributed over the assigned subcarriers. To solve the average number of needed subcarriers for a link, we need to explore all the possibilities of subcarrier assignment schemes, which is exponential to the total subcarrier number. For instance, to compute the average number of subcarriers to transmit r_{ij} at link $\overrightarrow{ij}$, all kinds of subcarrier combinations should be taken into consideration. For each combination, we need to check if the traffic demand is satisfied. The complexities are even higher in the *BS* and the *RS*s that support multiple link flows. Therefore, we measure the average channel quality g_{ij} across all subcarriers to determine the needed resource instead of using g_{ij}^k at each subcarrier. Based on the average channel quality over time (e.g. mostly recent 2 s), the average needed number for a link in the frequency selective fading channel can be obtained using almost the same method as that in the static channel. By comparing the total median resource of the nodes in different paths, the REQUEST node(s) can determine the relay path(s). As is mentioned earlier, if the links between the *BS* and the *SS*s, or between the *RS*s and the *SS*s, experience time-varying frequency selective channel fading. Only these links require the simplification when computing the average needed resource. Based on this simplified scheme, we avoid the prohibitive complexity of exhaustive search.

3.4. A unified model for resource allocation

We aim to explore the maximum throughput of end-to-end flows for a given traffic pattern. The allocable resources of the fixed-power scheduling are the subcarriers, while those of the dynamic power scheduling are both the subcarriers and the transmission powers. The throughput maximization problem can be converted to an equivalent problem of finding the maximum scaling factor. In this subsection, we formulate a unified optimization model for resource allocation in the OFDMA relay networks. Note that this unified model only characterizes the system objective and the essential resource constraints. The unified optimization problem is described as the following ($P1$):

$$\max \quad \theta \tag{9}$$

$$\text{s.t.} \quad r_{ij} \geqslant \theta\gamma_{ij}, \forall \overrightarrow{ij} \in \mathscr{L}, \tag{10}$$

$$\bigcup \Omega_{ij} \subseteq \mathscr{K}, \quad \forall \overrightarrow{ij} \in \mathscr{L}, \tag{11}$$

$$\Omega_{ij} \bigcap \Omega_{xy} = \varnothing, \quad \forall \overrightarrow{ij}, \overrightarrow{xy} \in \mathscr{L}, \tag{12}$$

$$\text{allocable power of node } i \leqslant P_i, \quad \forall i \in \mathscr{V}. \tag{13}$$

The physical meanings of the variables are explained in Table 1. The first constraint means that the throughput of all active links are evenly scaled up. The principles to compute r_{ij} are different in the fixed-power verses the dynamic power schedulers, the static tones verses the frequency selective tones, and the coding-oblivious verses the coding-aware schemes. The second constraint indicates that the union of the allocated subcarrier sets cannot not exceed the available subcarrier set $\mathscr{K}$. In the third constraint, a subcarrier cannot be assigned to more than one link at the same time. The fourth constraint is only applicable to the dynamic power scheduling, in which the allocable power of a node cannot exceed the corresponding maximum transmission power. In the following sections, we present the specific optimization models for each network scenario.

4. Scheduling with fixed power allocation

In this section, optimization models are formulated to maximize the network throughput for an arbitrary traffic pattern. We propose a *polynomial time* subcarrier assignment algorithm to exploit the benefits of both multiuser diversity and practical network coding.

4.1. Coding-oblivious scheduling with static tones

In order to simplify our analysis, we investigate the resource allocation problem starting from a **static** network. Because OFDMA has a centralized scheduling mechanism, the maximum throughput is achieved when the *BS* allocates subcarriers to the radio devices with favorable channel quality. Thus, we define the network capacity to be the maximum end-to-end throughput constrained by a fixed traffic pattern γ or the maximum scaling factor equivalently. The mathematical model to find the coding-oblivious network capacity is described by the following maximization ($P2$):

$$\max \quad \theta$$

$$\text{s.t.} \quad r_{i0} \geqslant \theta\gamma_{i0}, \quad \forall\, i \in \mathscr{U}_0, \tag{14}$$

$$r_{0i} \geqslant \theta\gamma_{0i}, \quad \forall\, i \in \mathscr{D}_0, \tag{15}$$

$$r_{im} \geqslant \theta\gamma_{im}, \quad \forall\, i \in \mathscr{U}_m, \tag{16}$$

$$r_{mi} \geqslant \theta\gamma_{mi}, \quad \forall\, i \in \mathscr{D}_m, \tag{17}$$

$$\sum_{i\in\mathscr{U}_0} x_{i0} + \sum_{i\in\mathscr{D}_0} x_{0i} + \sum_{m\in\mathscr{M}} \left(\sum_{i\in\mathscr{D}_m} x_{mi} + \sum_{i\in\mathscr{U}_m} x_{im} \right) \leqslant K, \tag{18}$$

where K is the total number of subcarriers and x_{ij} is the needed number of subcarriers at link $\overrightarrow{ij}$. The objective function θ is the scaling factor of the fixed traffic pattern. The constraints in Eqs. (14)–(17) represent the throughput requirements for a given scaling factor. The last constraint means that the assigned subcarriers have to be within the system resource of K subcarrriers.

Since the transmission power of a node is fixed and evenly distributed over all the subcarriers, the maximization (P2) is a simple linear programming problem. Thus, the scaling factor can be calculated by letting the inequality constraints be equality ones. The coding-oblivious scaling factor, θ_{co}, is expressed as

$$\theta_{co} = \frac{K}{\sum_{i\in\mathscr{D}_0} \frac{\gamma_{0i}}{c_{0i}} + \sum_{i\in\mathscr{U}_0} \frac{\gamma_{i0}}{c_{i0}} + \sum_{m\in\mathscr{M}} \left(\sum_{i\in\mathscr{U}_m} \frac{\gamma_{im}}{c_{im}} + \sum_{i\in\mathscr{D}_m} \frac{\gamma_{mi}}{c_{mi}} \right)}.$$

Solving the above equations may generate fractional subcarrier allocation, but the fractional value can be treated as the probability of occupying a subcarrier in a large time window.

4.2. Coding-aware scheduling with static tones

Next, we analyze system capacity of an OFDMA relay network when XOR coding is adopted by relay stations. According to the foregoing routing and XOR-coding operations, one can derive the coded flow vector γ_m^c and the remaining flow vector γ_m^r for all $m \in \mathscr{M}$. Denote r_{mi}^c and r_{mi}^r to be the throughput of the coded link and the remaining link at the relay station m, respectively. Let x_{mi}^c and x_{mi}^r be the number of allocated subcarriers for r_{mi}^c and r_{mi}^r. When the XOR-coding is adopted, the mathematical model of maximizing network throughput can be expressed as ($P3$):

$$\max \quad \theta$$

$$\text{s.t.} \quad r_{mi}^c \geqslant \theta\gamma_{mi}^c, \quad \forall\, i \in \mathscr{D}_m^c, \tag{19}$$

$$r_{mi}^r \geqslant \theta\gamma_{mi}^r, \quad \forall\, i \in \mathscr{D}_m^r. \tag{20}$$

Eqs. (14)–(16),

$$\sum_{m\in\mathscr{M}} \left(\sum_{i\in\mathscr{U}_m} x_{im} + \sum_{i\in\mathscr{D}_m^c} x_{mi}^c + \sum_{i\in\mathscr{D}_m^r} x_{mi}^r \right) + \sum_{i\in\mathscr{U}_0} x_{i0} + \sum_{i\in\mathscr{D}_0} x_{0i} \leqslant K. \tag{21}$$

Similar to the coding-oblivious scheme, the optimal scaling factor of coding-aware scheme can be computed by

$$\theta_{ca}=\frac{K}{\sum_{i\in\mathscr{D}_0}\frac{\gamma_{0i}}{c_{0i}}+\sum_{i\in\mathscr{U}_0}\frac{\gamma_{i0}}{c_{i0}}+\sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{U}_m}\frac{\gamma_{im}}{c_{im}}+\sum_{i\in\mathscr{D}_m^c}\frac{\gamma_{mi}^c}{c_{mi}^c}+\sum_{i\in\mathscr{D}_m^r}\frac{\gamma_{mi}^r}{c_{mi}^r}\right)}.$$

Then, network coding gain can be expressed as,

$$\frac{\theta_{ca}}{\theta_{co}}=1+\frac{\sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{D}_m}\frac{\gamma_{mi}}{c_{mi}}-\sum_{i\in\mathscr{D}_m^c}\frac{\gamma_{mi}^c}{c_{mi}^c}-\sum_{i\in\mathscr{D}_m^r}\frac{\gamma_{mi}^r}{c_{mi}^r}\right)}{\sum_{i\in\mathscr{D}_0}\frac{\gamma_{0i}}{c_{0i}}+\sum_{i\in\mathscr{U}_0}\frac{\gamma_{i0}}{c_{i0}}+\sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{U}_m}\frac{\gamma_{im}}{c_{im}}+\sum_{i\in\mathscr{D}_m^c}\frac{\gamma_{mi}^c}{c_{mi}^c}+\sum_{i\in\mathscr{D}_m^r}\frac{\gamma_{mi}^r}{c_{mi}^r}\right)}. \quad (22)$$

Obviously, with fixed power allocation, the network coding gain becomes larger if the traffic loads of direct *BS–SS* communication decrease. To analyze the impact of physical parameters on the network coding gain, we consider a simple two-way relay network shown in Fig. 1a and b. When both *BS* and *SS* relay their traffic through *RS*, the link traffic pattern satisfies $\gamma_{br}=\gamma_{rs}$ and $\gamma_{sr}=\gamma_{rb}$. Assume that the downlink traffic is no less than the uplink traffic, i.e. $\gamma_{br}\geqslant\gamma_{sr}$. The coding gain of the two-way relay network can be expressed as

$$\frac{\theta_{ca}}{\theta_{co}}=1+\frac{\gamma_{rs}/c_{rs}+\gamma_{rb}/c_{rb}-\gamma_{rb}/\min\{c_{rs},c_{rb}\}-(\gamma_{rs}-\gamma_{rb})/c_{rs}}{\gamma_{br}/c_{br}+\gamma_{sr}/c_{sr}+\gamma_{rb}/\min\{c_{rs},c_{rb}\}+(\gamma_{rs}-\gamma_{rb})/c_{rs}}. \quad (23)$$

The relationships between network coding gain and system parameters are summarized as below:

Lemma 1. *Let the traffic pattern and the transmission powers be tunable, we have the following properties on the two-way relay network:*

- *The optimal coding gain is achieved when the downlink/uplink traffic ratio is 1.*
- *The coding gain is an increasing function of the transmission power of the base station or the subscriber.*
- *The coding gain is a decreasing function of the transmission power of the relay station when channel qualities of the BS and SS are the same at the RS side.*

Proof. Please refer to Appendix A. □

4.3. Scheduling with frequency selective channel fading

We have analyzed the performance of coding-oblivious and coding-aware schemes in static networks so far. However, wireless channels are frequency selective because of the multi-path propagation. Different subcarriers of a radio device may experience different attenuations. Though the relay decision is based on the "average" channel quality, one can still take advantage of multiuser diversity in each time slot. After the forwarding paths are determined, the channel-aware scheduling problem without XOR-coding can be expressed as $(P4)$:

$$\begin{aligned}&\max\quad \theta\\ &\text{s.t.}\quad \lambda_{ij}^k\in\{0,1\},\quad \forall\ \overrightarrow{ij}\in\mathscr{L},\ k\in\mathscr{K}, \qquad (24)\\ &\text{Eqs. (14)–(17)},\\ &\sum_{i\in\mathscr{U}_0}\lambda_{i0}^k+\sum_{i\in\mathscr{D}_0}\lambda_{0i}^k+\sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{U}_m}\lambda_{im}^k+\sum_{i\in\mathscr{D}_m}\lambda_{mi}^k\right)\leqslant 1,\ \forall k\in\mathscr{K}, \qquad (25)\end{aligned}$$

where λ_{ij}^k is the subcarrier allocation indicator such that $\lambda_{ij}^k=1$ if and only if subcarrier k is assigned to the link $\overrightarrow{ij}$. In the objective function of problem $(P4)$, the data rate of link $\overrightarrow{ij}$ is

$$r_{ij}=\sum_{k=1}^{K}\lambda_{ij}^k c_{ij}^k=\sum_{k=1}^{K}\lambda_{ij}^k\cdot W\log_2\left(1+\frac{P_i g_{ij}^k}{K}\right). \quad (26)$$

When XOR-coding is introduced in the frequency selective OFDMA relay networks, $(P3)$ is transformed into the following optimization problem $(P5)$:

$$\begin{aligned}&\max\quad \theta\\ &\text{s.t.}\quad \lambda_{ij}^k\in\{0,1\},\quad \forall\ \overrightarrow{ij}\in\mathscr{L},\ k\in\mathscr{K}. \qquad (27)\\ &\qquad \text{Eqs.}(14)–(16),\ (19),\ (20),\\ &\qquad \sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{D}_m^c}\lambda_{mi}^{c,k}+\sum_{i\in\mathscr{D}_m^r}\lambda_{mi}^{r,k}+\sum_{i\in\mathscr{U}_m}\lambda_{im}^k\right)\\ &\qquad\quad +\sum_{i\in\mathscr{U}_0}\lambda_{i0}^k+\sum_{i\in\mathscr{D}_0}\lambda_{0i}^k\leqslant 1, \qquad (28)\end{aligned}$$

where $\lambda_{mi}^{c,k}$ and $\lambda_{mi}^{r,k}$ denote the subcarrier allocation indicators for the coded link $\overrightarrow{ij}$ and the remaining link $\overrightarrow{ij}$, respectively. The throughput of coded and remaining links can also be computed by using Eq. (26).

The optimal subcarrier assignment for OFDMA systems is generally very difficult to solve because it may involve a large-scale mixed integer programming problem. Let L_{tot} be the total number of link flows in a coding-aware OFDMA relay network, there are $(L_{tot})^K$ possibilities to allocate subcarriers. The optimal scaling factor can be found through an exhaustive search of all $(L_{tot})^K$ subcarrier assignment schemes. Owing to this prohibitive complexity, we present a heuristic channel-aware subcarrier assignment scheme that is extended from the downlink scheduler in [9,10]. The basic idea is to assign a favorable subcarrier to the link with the smallest scaling factor in each round. After the link is determined, it is assigned the best subcarrier in the available subcarrier set. Because the coding-oblivious and the coding-aware assignment schemes are very similar, we only discuss the latter. Denote Ω_l to be the set of subcarriers assigned to link l for $l\in\mathscr{L}$, the detailed algorithm is specified in Fig. 8. The proposed algorithm is composed of the initialization procedure and the assignment of remaining subcarriers. In the initialization procedure, the scheduler assigns subcarriers to the mobile stations first. We also examine the complexity of the proposed subcarrier assignment scheme. In each step, the scheduler needs L_{tot} comparisons to identify the link of the smallest weighted throughput. At most K additional comparisons are required to search the subcarrier with the best throughput for the selected link. Thus, the total computational complexity is bounded by $K(L_{tot}+K)$. The presented algorithm can be further improved at the cost of increased complexity in the special scenario where some of links experience only flat fading. The scheduler can assign subcarriers to the links with frequency selective fading first, and shift the less important subcarriers of the frequency selective links to the links with flat fading until the scaling factor is maximized.

Subcarrier Assignment

Initialization

```
Construct link set 𝓛 after XOR coding and
      let 𝓛̃ = 𝓛; Ω_l = ∅, ∀l ∈ 𝓛;
𝒦 = {1, 2, ··· , K};
for l ∈ 𝓛̃, k = 1 to K
   Compute the per-subcarrier power of link l: p_l;
   k* = arg max_k g_l^k;
   𝒦 = 𝒦 \ {k*}, Ω_l = {k*};
   𝓛̃ = 𝓛̃ \ {l};
end for
```

Assign Remaining Subcarriers

```
While 𝒦 ≠ ∅,
   Find l* that has the smallest r_l/γ_l, ∀l ∈ 𝓛, γ_l ∈ γ;
   Find k* that has g_{l*}^{k*} ≥ g_{l*}^k, ∀k*, k ∈ 𝒦;
   Ω_{l*} = Ω_{l*} ∪ {k*}, 𝒦 = 𝒦 \ {k*};
   r_{l*} = r_{l*} + W log_2(1 + p_{l*} g_{l*}^{k*});
End While
```

Fig. 8. Subcarrier assignment algorithm with fixed power allocation.

5. Scheduling with dynamic power allocation

In this section, we model the optimal resource allocation as a nonlinear integer programming problem and present an efficient technique to jointly allocate subcarrier and transmission power for radio devices. Our study shows that network coding can bring both the coding gain and the power gain.

5.1. Observation of power gain

An important advantage of OFDMA is that the transmission power can be assigned on each subcarrier dynamically so as to increase channel SNR. The dynamic power can potentially bring more throughput gain in the presence of network coding. The reason lies in that the XOR coding saves two kinds of network resource: subcarriers and the corresponding transmission power. The saved power is shifted to the allocated subcarriers and further reduces the needed subcarriers to transmit other link flows. Here, we illustrate this interesting property. According to Eq. (5), the relationship between throughput and subcarrier number is shown in Fig. 9. Assume that a *RS* has two encodable flows, each has a traffic load of *C*. The *RS* is also assumed to have the same channel gains to both receivers. Without network coding, the *RS* needs x_{rs} to transmit 2*C*. While it only needs x'_{rs} to transmit the encoded flow with a load of *C*. Due to the concavity of the subcarrier/throughput curve, XOR coding can save more than half of the subcarriers. While only half of the subcarriers are saved if the transmission power is fixed on each subcarrier. Obviously, one can see that at the coding node, the reasons for saving subcarriers are not only the reduced traffic loads, but also the dynamic power transfer. We name this extra performance gain as the "power gain". Because it is very difficult to quantify the "coding gain" and the "power gain" separately, we use a new term "coding and power gain" to denote the ratio of the coding-aware scaling factor to the coding-oblivious scaling factor in the OFDMA relay networks with dynamic power distribution.

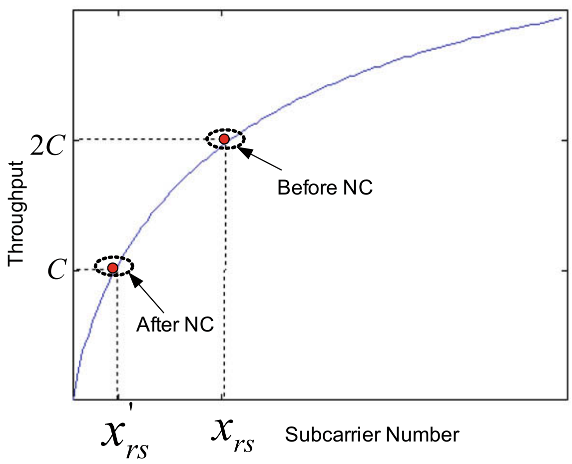

Fig. 9. An illustration of the coding and power gain.

5.2. System model of coding-aware scheduling

We consider the joint subcarrier assignment and power allocation for a general OFDMA relay network with frequency selective channel fading. Each subscriber has at most a single uplink traffic that may use a quantity of subcarriers. The power of a transmitter is distributed on the allocated subcarriers. Under the power constraints, the design objective is to maximize the scaling factor θ. When XOR coding scheme is adopted, the mathematical model of joint subcarrier and power allocation can be expressed as (*P*6):

$$\max \quad \theta$$

$$\text{s.t.} \quad \lambda_{ij}^k \in \{0,1\}, \quad \forall\ \overrightarrow{ij} \in \mathscr{L},\ k \in \mathscr{K},$$

$$\text{Eqs.}(14){-}(16),\ (19),\ (20),$$

$$\sum_{m\in\mathscr{M}}\left(\sum_{i\in\mathscr{D}_m^c}\lambda_{mi}^{c,k}+\sum_{i\in\mathscr{D}_m^r}\lambda_{mi}^{r,k}+\sum_{i\in\mathscr{U}_m}\lambda_{im}^{k}\right)+\sum_{i\in\mathscr{U}_0}\lambda_{i0}^{k}+\sum_{i\in\mathscr{D}_0}\lambda_{0i}^{k}\leqslant 1,$$

$$\sum_{k\in\mathscr{K}} p_{i0}^k \leqslant P_i, \quad \forall i \in \mathscr{U}_0, \tag{29}$$

$$\sum_{k\in\mathscr{K}} p_{im}^k \leqslant P_i, \quad \forall i \in \mathscr{U}_m, \tag{30}$$

$$\sum_{i\in\mathscr{D}_0}\sum_{k\in\mathscr{K}} p_{0i}^k \leqslant P_0, \tag{31}$$

$$\sum_{k\in\mathscr{K}}\left(\sum_{i\in\mathscr{D}_m^c} p_{mi}^{c,k}+\sum_{i\in\mathscr{D}_m^r} p_{mi}^{r,k}\right)\leqslant P_m, \quad \forall m \in \mathscr{M}, \tag{32}$$

where the throughput of link $\overrightarrow{ij}$ is computed by the following equation:

$$r_{ij} = \sum_{k=1}^{K} \lambda_{ij}^{k} \cdot W\log_2\left(1 + p_{ij}^{k} g_{ij}^{k}\right). \quad (33)$$

Compared with the preceding optimization models, there are four additional constraints in Eqs. (29)–(32). For each transmitting node, the total transmission power across all the subcarriers cannot violate the power budget. Likewise, a subcarrier cannot be assigned to more than one link in every time slot. The throughput of the link flows should be greater than the scaled traffic ratios.

5.3. Subcarrier assignment

Due to the difficulty to find the optimal solution of ($P6$), we present a low-complexity algorithm to assign subcarriers based on [9,10]. Even the suboptimal scheme with dynamic power allocation is not straightforward at all. The difficulty is that the throughput of a link is not fixed at a subcarrier, but depends on the assigned transmission power. The throughput increment of this link is diminishing with the increase of allocated subcarriers. We assume that the transmission power is evenly, but dynamically distributed over all the allocated subcarriers. The basic idea of the proposed assignment scheme is similar to that with fixed power allocation. But there is one major **difference**. When a node is allocated a new subcarrier, the power per-subcarrier changes and the throughput of all the links in that node should be recomputed. The detailed algorithm is shown in Fig. 10. Likewise, the scheduler assigns subcarriers to the mobile stations first in the initialization procedure. It is worthwhile to compare the complexity of the proposed schemes with fixed and dynamic power allocations. When assigning a subcarrier, the scheduler needs L_{tot} comparisons to find the link with the smallest weighted throughput. It also needs at most K comparisons to find the best subcarrier of that link. Once a new subcarrier is assigned, the transmitting node recalculates the throughput of each assigned subcarrier and the throughput of the affiliated links. The rate updating needs at most K operations. Hence, the total computational complexity of the subcarrier assignment algorithm is bounded by $K(L_{tot} + 2K)$. Because the number of subcarriers is larger than that of links, the complexities of the scheduling algorithms with fixed and dynamic power allocations are both approximated by $\mathbf{O}(K^2)$.

5.4. Adaptive power allocation

In the last subsection, we propose a polynomial time assignment scheme to scale the throughput of a given traffic pattern. The scheduling factor can be further improved if the total transmission power is adaptively distributed

Subcarrier Assignment

Initialization

$\mathcal{K} = \{1, 2, \cdots, K\}$; $\Omega_{ij} = \Omega_i = \varnothing, \ \forall i, j, \in \mathcal{V}, k \in \mathcal{K}$;
$\lambda_{ij}^{k} = 0$; $r_{ij} = 0, \forall i, j, \in \mathcal{V}, k \in \mathcal{K}$;
Active Link Set:
  $\mathcal{L} = \{\overrightarrow{ij} \mid i, j, \in \mathcal{V}, \text{ and } \gamma_{ij}, \gamma_{ij}^{c}, \gamma_{ij}^{r} \text{ are all nonzero}\}$;
  // The active link set also includes the encoded
  //and the remaining links.
$\widetilde{\mathcal{L}} = \mathcal{L}$;
for $\overrightarrow{ij} \in \widetilde{\mathcal{L}}$ & $\widetilde{\mathcal{L}} \neq \varnothing$
  $k^* = \arg\max_k g_{ij}^{k}$;
  $\mathcal{K} = \mathcal{K} \setminus \{k^*\}$, $\Omega_{ij} = \{k^*\}$, $\Omega_i = \Omega_i \cup \{k^*\}$;
  // Record the subcarrier assignment for the links
  // and the transmitting nodes;
  $\widetilde{\mathcal{L}} = \widetilde{\mathcal{L}} \setminus \{\overrightarrow{ij}\}$;
end for

Assign Remaining Subcarriers

While $\mathcal{K} \neq \varnothing$**,**
  Find $\overrightarrow{ij}^*$ that has the smallest $\frac{r_{ij}}{\gamma_{ij}}$, $\forall \overrightarrow{ij} \in \mathcal{L}, \gamma_{ij} \in \boldsymbol{\gamma}$;
  Find k^* that has $g_{ij^*}^{k^*} \geq g_{ij^*}^{k}$, $\forall k^*, k \in \mathcal{K}$;
  $\Omega_{ij^*} = \Omega_{ij^*} \cup \{k^*\}$, $\Omega_{i^*} = \Omega_{i^*} \cup \{k^*\}$, $\mathcal{K} = \mathcal{K} \setminus \{k^*\}$;
  $p_{i^*} = P_{i^*}/|\Omega_{i^*}|$;
  // update the power per-subcarrier for the node i^*;
  Update the throughput of every subcarrier in Ω_{i^*};
  Calculate the throughput of outgoing links at node i^*;
End While

Fig. 10. Subcarrier assignment algorithm with dynamic power.

over every subcarrier. After the subcarrier assignment is complete, the original coding-aware and channel-aware optimization model ($P6$) can be decomposed into a set of sub-optimization problems in the transmitting nodes.

- For an uplink subscriber $i \in \mathscr{U}_0$, the sub-optimization problem ($P7$) is:

$$\max \sum_{k\in\Omega_{i0}} W\log_2\left(1 + p_{i0}^k g_{i0}^k\right) \tag{34}$$

$$\text{s.t.} \sum_{k\in\Omega_{i0}} p_{i0}^k \leqslant P_i. \tag{35}$$

- For an uplink subscriber $i \in \mathscr{U}_m$, the sub-optimization problem ($P8$) is:

$$\max \sum_{k\in\Omega_{im}} W\log_2\left(1 + p_{im}^k g_{im}^k\right) \tag{36}$$

$$\text{s.t.} \sum_{k\in\Omega_{im}} p_{im}^k \leqslant P_i. \tag{37}$$

- For the base station 0, the sub-optimization problem ($P9$) is:

$$\begin{aligned} \max\ & \theta \\ \text{s.t.}\ & \sum_{i\in\mathscr{D}_0}\sum_{k\in\Omega_{0i}} p_{0i}^k \leqslant P_0, \\ & r_{0i} \geqslant \theta\gamma_{0i}, \quad \forall i \in \mathscr{D}_0. \end{aligned} \tag{38}$$

- For a relay station m, the sub-optimization problem ($P10$) is:

$$\begin{aligned} \max\ & \theta \\ \text{s.t.}\ & \sum_{i\in\mathscr{D}_m^c}\sum_{k\in\Omega_{mi}^c} p_{mi}^{c,k} + \sum_{i\in\mathscr{D}_m^r}\sum_{k\in\Omega_{mi}^r} p_{mi}^{r,k} \leqslant P_m, \\ & r_{mi}^c \geqslant \theta\gamma_{mi}^c, \quad \forall i \in \mathscr{D}_m^c, \qquad (39) \\ & r_{mi}^r \geqslant \theta\gamma_{mi}^r, \quad \forall i \in \mathscr{D}_m^r. \qquad (40) \end{aligned}$$

The above decomposition is performed for each transmitting node. For an uplink subscriber, there is only a single link so that the objective is to maximize the link throughput. In terms of the *BS*, it has a number of links whose throughput are lower-bounded by the scaled values. The resource allocation model of a *RS* is very close to that of the *BS* except that the *RS* may have both encoded links and the remaining links. The decomposition is optimal after the subcarrier assignment is finished in the original problem. In the original problem ($P6$), the goal of power allocation is to find the best multiplier to scale up the traffic pattern. While in the sub-problems, the goal of power allocation is to find the best scaling factors for the transmitting nodes locally. Thus, the scaling factor of ($P6$) is exactly the minimum one among all the sub-problems. In our simulation, we measure the effective throughput computed by the product of the scaling factor in ($P6$) and the traffic pattern.

In the above models, ($P7$) and ($P8$) are merely sub-problems of ($P9$) and can be easily solved through a water-filling algorithm. Besides, the maximization ($P10$) is very similar to ($P9$). Hence, we only look at the solution technique of adaptive power allocation problem ($P9$) at the *BS*. The constraints in Eq. (38) are not concave functions of the transmission powers when we compute r_{0i} through Eq. (1). The maximization ($P9$) is in fact a nonlinear non-convex optimization problem, which is hard to obtain the optimal solution. Thus, we consider the simplified power allocation problem that has the close-form power distribution. The power budget of a link is proportional to the number of allocated subcarriers in a transmitting node. Given the power budget, the *BS* determines the total transmission power for each downlink flow. When the total power of a link has been assigned, it can decide how to distribute the power over each allocated subcarrier for a link.

5.4.1. Intra-link power allocation

Here, we consider the simplified power allocation problem that has close-form power distribution. The power budget of a link is proportional to the number of allocated subcarriers. Given the power budget, the link can adaptively assign the transmission power over the allocated subcarrier set.

Define a vector $\mathbf{p}_0 = \{p_{01}, p_{02}, \ldots, p_{0,D_0}\}$ to be the set of total power for all links at the *BS*, where $p_{0i} = \frac{|\Omega_{0i}|}{|\Omega_0|}P_0$ for all $i \in \mathscr{D}_0$. We assume that the subcarriers $\{1, 2, \ldots, |\Omega_{0i}|\}$ are allocated for the link $\overrightarrow{0i}$ and the channel qualities satisfy $g_{0i}^1 \leqslant g_{0i}^2 \cdots\ g_{0i}^{|\Omega_{0i}|}$. The intra-link power allocation problem can be solved by the standard water-filling algorithm in [10]. For the link $\overrightarrow{0i}$, there has

$$p_{0i}^k = p_{0i}^1 + \frac{g_{0i}^k - g_{0i}^1}{g_{0i}^k \cdot g_{0i}^1}, \tag{41}$$

$$p_{0i} = \sum_{k\in\Omega_{0i}} p_{0i}^k = |\Omega_{0i}| \cdot p_{0i}^1 + \sum_{k=2}^{|\Omega_{0i}|} \frac{g_{0i}^k - g_{0i}^1}{g_{0i}^k \cdot g_{0i}^1}. \tag{42}$$

The above water-filling scheme achieves an optimal power allocation for a number of subcarriers allocated to a link. When the channel states are static and uniform, $p_{0i}^k = p_{0i}^1$ for all $k \in \Omega_{0i}$, and we have the following corollary:

Corollary 1. *Assigning transmission power evenly over the allocated subcarriers is throughput optimal for a link with homogeneous channel qualities.*

6. Performance evaluation

6.1. Simulation setup

To evaluate the performance of proposed algorithms, we consider various network scenarios with both static and frequency selective wireless channels. The comparison criteria is to what extent our coding-aware routing and scheduling algorithms can scale up the throughput of a traffic pattern. Note that the end-to-end throughput is measured based on the smallest scaling factor. We simulate OFDMA relay networks with 256 subcarriers over a 3.5 MHz frequency band. The power density of additive gaussian noise is −174 dBm/Hz. The center frequency is 1 GHz that is usually chosen for broadband wireless communication systems. The path-loss exponent is set to 3.5 according to the ITU recommendation. The path-loss at a reference distance $d_0 = 1$ m is computed by $\left(\frac{\phi}{4\pi f d_0}\right)^2$ where f is the center frequency and ϕ is the speed of light in m/s.

Each transmitter is synchronized in a time slot so that the narrow-band tones are orthogonal. The desired BER of the MQAM modulation is 10^{-3}. The default time slot is 10 ms. The remaining tunable parameters are configured in each set of experiments.

6.2. Static two-way relay network

In this set of experiments, we first investigate the impacts of traffic pattern and transmission power on the end-to-end throughput and the coding gains of a static two-way relay network in Fig. 1. Then, the performance of coding-oblivious and coding-aware algorithms are compared for a multi-path two-way relay network in Fig. 2.

6.2.1. Impact of traffic pattern

Three nodes are evenly apart by 1000 m in a straight line. The transmission power of all nodes are uniformly 33 dbm ($\sim$ 2 W). When the traffic ratio of downlink over uplink, $\frac{\gamma_{bs}}{\gamma_{sb}}$, increases from 0.2 to 5, the evolution of throughput gain is illustrated in Fig. 11. One can see that the optimal coding gains are obtained at the point $\frac{\gamma_{bs}}{\gamma_{sb}} = 1$ for both fixed and dynamic power allocations. The throughput gain of the fixed power scheme is bounded by 1/3, while the dynamic power scheme has a throughput gain of over 50%. This is because the dynamic power allocation brings the power gain besides the coding gain.

6.2.2. Impact of transmission powers

In general, the transmission powers of wireless nodes are not equal. For example, the *BS* and the *RS*s might have larger transmission powers compared with the *SS*s. This set of experiments consider two cases: (i) *BS* with various power levels; (ii) *RS* with various power levels. The uplink and downlink have the identical traffic loads. In the first scenario, the transmission powers of the *RS* and the *SS* are set to be 33 dbm and the power of *BS* increases from 25 dbm to 45 dbm. Fig. 12 shows that larger *BS* power results in larger throughput for all the schems. Fig. 13 shows that the throughput gains increase along with the growth

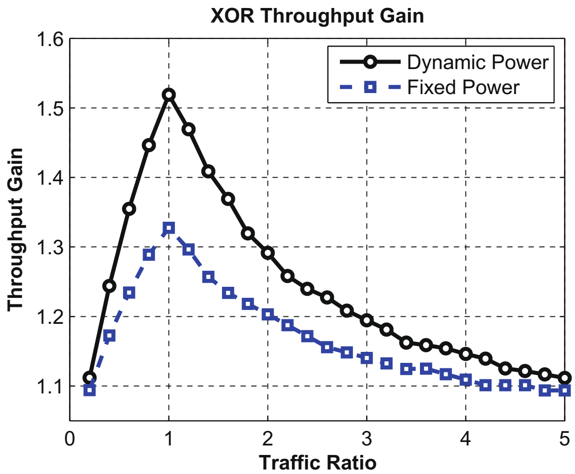

Fig. 11. Traffic ratio VS throughput gain.

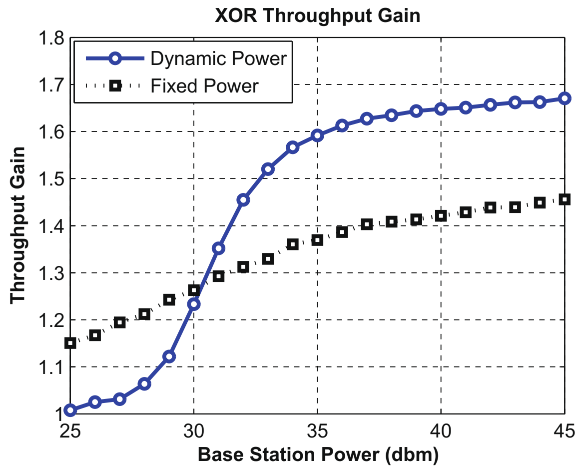

Fig. 13. *BS* power VS throughput gain.

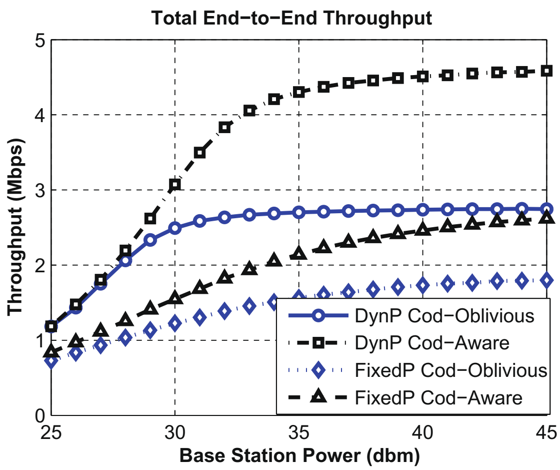

Fig. 12. *BS* power VS aggregate end-to-end throughput.

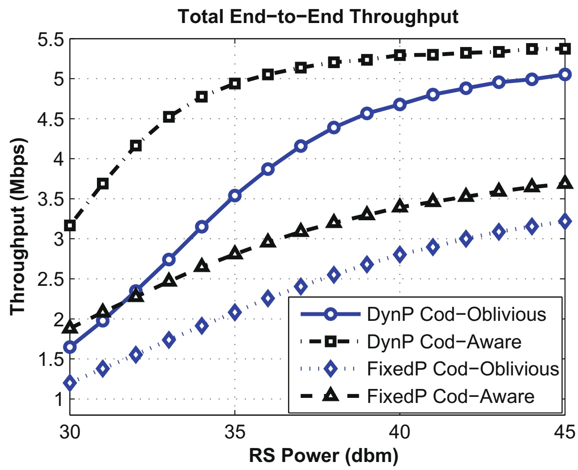

Fig. 14. *RS* power VS aggregate end-to-end throughput.

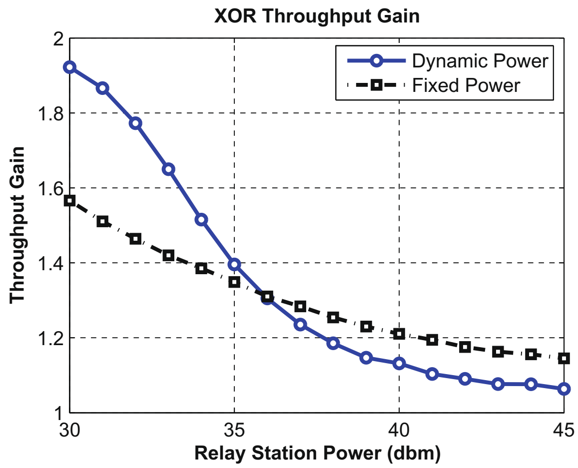

Fig. 15. *RS* power VS throughput gain.

of the *BS*'s power. Comparing the fixed power scheme with the dynamic power scheme, one can see that the latter brings a much larger throughput gain when the base station's transmission power is high. In the second scenario, the transmission powers of the *BS* and the *SS* are set to be 40 dbm and 33 dbm, respectively. The transmission power of the *RS* increases from 30 dbm to 45 dbm. Figs. 14 and 15 illustrate the aggregate throughput and the coding gains of the proposed schedulers. When the transmission power of *RS* increases, it saves less subcarriers, resulting in the decreased network coding gains.

6.2.3. Impact of routing

This set of experiments explain why the encodable flows should choose/update routing simultaneously. The traffic loads are the same in the uplink and downlink. The transmission power is fixed and evenly distributed over the entire band. In Fig. 2, the *SS* is 2000 m away from the *BS*. There are two relay stations in the middle, where the hop distances $BS - RS_1, RS_1 - SS, BS - RS_2$ and $RS_2 - SS$ are 1400 m, 900 m, 1200 m, 1200 m, respectively. The transmission powers of *RS*s and *SS*s are set to be 33 dbm($\sim$ 2 W). We investigate the throughput performance by changing the *BS*'s power. The end-to-end throughput on six selective paths are compared in Fig. 16. Let us focus on the end-to-end throughput when the BS power is greater than 35dbm. When the XOR-coding is adopted, the downlink and uplink flows can benefit from choosing the same node to relay their traffic. As is shown in Fig. 16, the throughput in the path set $\{BS \rightarrow RS_2 \rightarrow SS, SS \rightarrow RS_2 \rightarrow BS\}$ is greater than that in the set $\{BS \rightarrow RS_1 \rightarrow SS, SS \rightarrow RS_1 \rightarrow BS\}$. Thus, it is important to properly choose the relay station for XOR-coding. The proposed routing strategy in Section 3 compares the subcarrier consumption over two paths and select $\{BS \rightarrow RS_2 \rightarrow SS, SS \rightarrow RS_2 \rightarrow BS\}$ to relay the traffic. Next, we will show that unilateral path selection of a flow might lead to the inefficient routing. When network coding is not used, the best throughput is obtained in the path set $\{BS \rightarrow RS_1 \rightarrow SS, SS \rightarrow RS_2 \rightarrow BS\}$ and the worst one is obtain in the path set $\{BS \rightarrow RS_2 \rightarrow SS, SS \rightarrow RS_1 \rightarrow BS\}$. This means that the downlink flow prefers RS_1 and the uplink flow is inclined to RS_2. If the downlink flow enters in the network first, it will pick the path $BS \rightarrow RS_1 \rightarrow SS$. When the uplink flow joins later, it compares the subcarrier consumption of both paths and select RS_1 due to the benefit from network coding. Hence, the unilateral path selection might cause performance loss. Besides, the unilateral path selection also leads to different types of inefficient routings in some other scenarios. We omit the analysis since they are similar to the above case study.

6.3. Suboptimal algorithms verses optimal algorithms

The optimal scheduler is found throughput the exhaustive search method. Since the simulation platform can pro-

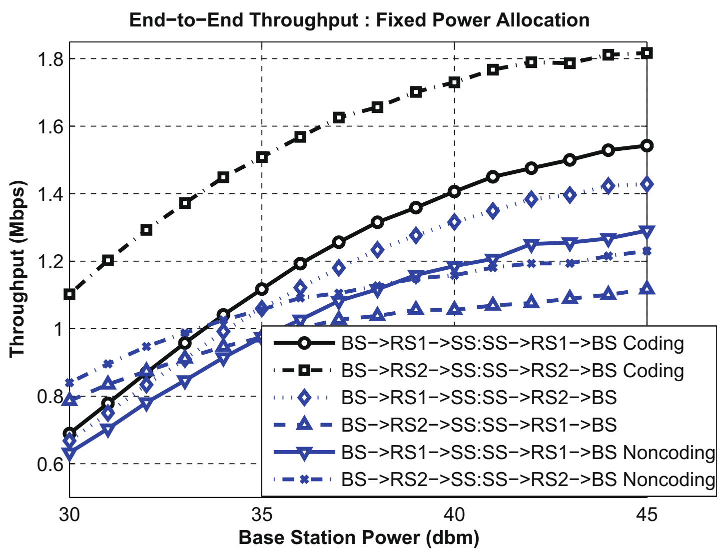

Fig. 16. Impact of coding-aware relay selection.

cess only over 4000 schedulers per-second, we only consider a small number of subcarriers and links in order to reduce the time to find the optimum. Note that the performance gap between the heuristics assignment and the optimum is affected by many factors such as the transmission powers, channel gains, the number of subcarriers and the number of links. The frequency band is divided into 256 orthogonal subcarriers, in which 12 of them are used in the simulations. Two types of network topology are considered, in which the first one is an OFDMA downlink network, and the second one is the OFDMA relay network shown in Fig. 1. The channel over each link is modeled as a frequency selective channel consisting of five independent propagation paths. The received signal fading on the first tap is characterized by a Ricean distribution with K-factor equal to 1. The fading on the other four taps follows a Rayleigh distribution. The maximum Doppler shift of each propagation tap is 5 Hz. We generate the channel gains over 2000 realizations based on Stanford University Interim (SUI)-3 and the Rayleigh fading codes in [10]. The relative powers of the other four Rayleigh multipath components are $[-8.6859\ \ -17.3718\ \ -26.0577\ \ -34.7436]$dB.

In the first scenario, the distances between *BS* and *SS*s are within the range [600 m, 1400 m]. The total transmission power of *BS* is 5 W. For the fixed-power scheduling, the power per-subcarrier is 19.5 mW. For the dynamic power scheduling, the transmission power of these 12 subcarriers is 234.4 mW in total. We compare the normalized throughput between the optimal algorithms and the heuristic algorithms in Figs. 17 and 18. Note that the normalized throughput are averaged over 10 slots. The digits below the marks represent the traffic ratios. Fig. 17 shows that the performance of the heuristic algorithm is very close to the optimum when the power is fixed over subcarriers. Fig. 18 also evaluates the efficiency of the heuristic algorithm with dynamic power allocation. The gaps between the heuristic results and the corresponding optimums are acceptable.

In the second scenario, the OFDMA relay network contains four links. To find the optimum, we need to validate 16777216 assignment strategies in one slot. The distance between *BS* and *RS* is 1000 meters and that between *RS* and *SS* is 600 m. The total powers of *BS*, *RS* and *SS* are 5 W, 2 W and 1 W, respectively. Because we only use 12

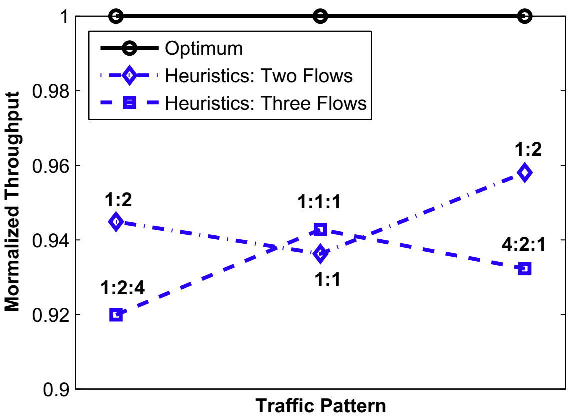

Fig. 17. Heuristics verses optimum: fixed power scheduling.

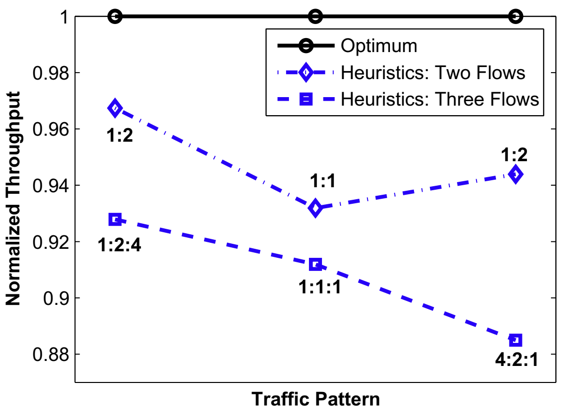

Fig. 18. Heuristics verses optimum: dynamic power scheduling.

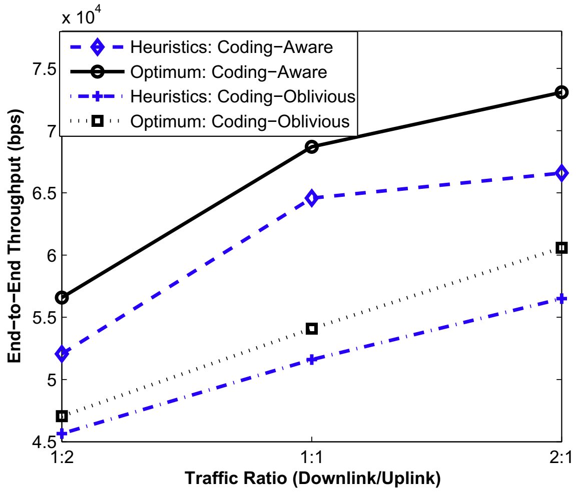

Fig. 19. Heuristics verses optimum: fixed power scheduling.

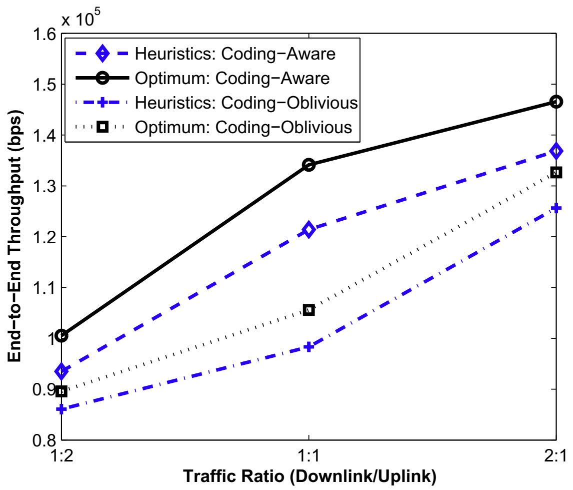

Fig. 20. Heuristics verses optimum: dynamic power scheduling.

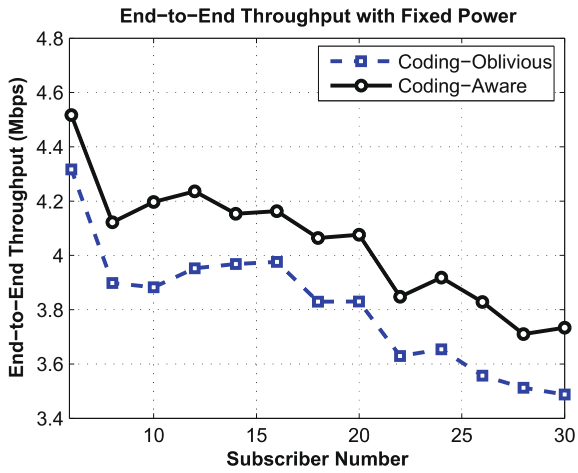

Fig. 21. Subscriber number VS end-to-end throughput: fixed power allocation.

subcarriers, the total powers of these 12 subcarriers are 234.4 mW, 93.8 mW and 46.9 mW in *BS*, *RS* and *SS*. Three traffic patterns are evaluated for both the fixed-power and the dynamic power schedulers. The throughput ratios of downlink/uplink are 0.5, 1, 2, respectively. Fig. 19 compares the end-to-end throughput of the heuristic algorithm with the optimal algorithm when the transmission powers are fixed. The performance gaps, shown in Fig. 19, are around 10% in the simple relay network. When the transmission powers are evenly but dynamically distributed over the assigned subcarriers, the end-to-end throughput are demonstrated in Fig. 20. The throughput gaps between the heuristic assignments and the optimums at this slot are around 10% in this set of experiments.

6.4. Random subscriber distribution with frequency selective fading

The performance of an OFDMA network with random subscriber distribution is evaluated in a 5000 × 5000 m square area. There are one *BS*, six *RS*s and a number of *SS*s. The *BS* is located in the center of the square area and the *RS*s are located at the vertices of a hexagon that are 1000 m away from the center *BS*. The *SS*s are randomly distributed in a circular area 300–1800 m far from the *BS*, which means only some of them need relaying. The circular area between the *SS*s and the *RS*s is more than three times the area between the *BS* and the *RS*s. Thus, about $\frac{1}{3} \sim \frac{1}{4}$ of total *SS*s on average do not need relay service. Because we consider random subscriber distribution, in some cases, the *SS*s are located in the smaller circular area between the *RS*s and the *BS*. Thus, there will not have XOR coding opportunities, hence the XOR coding gain. In this subsection, the wireless links experience slow frequency selective fading such that the channel is stationary (channel gain is constant) within each OFDM frame. To reduce the simulation burden, the route discovery schemes are triggered every 5 s and the paths are decided based on the average channel gains as well as traffic loads during that interval. In the first set of experiments, we compare

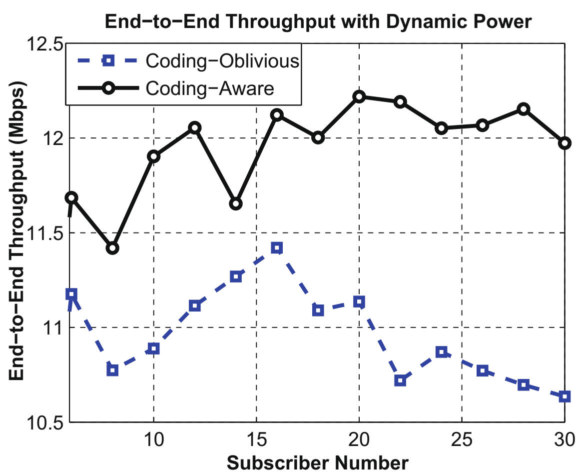

Fig. 22. Subscriber number VS end-to-end throughput: dynamic power allocation.

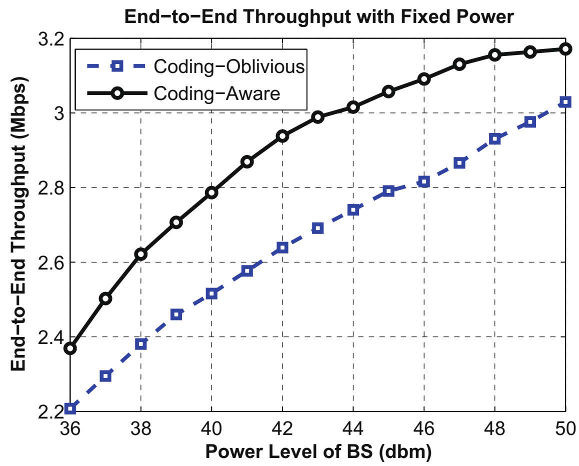

Fig. 23. BS power VS end-to-end throughput: fixed power allocation.

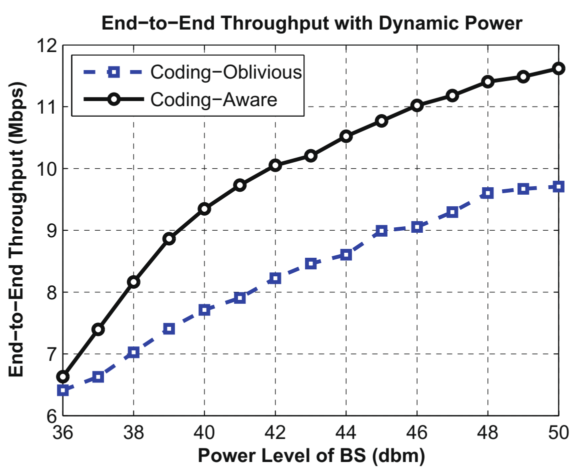

Fig. 24. BS power VS end-to-end throughput: dynamic power allocation.

the end-to-end throughput of coding-oblivious and coding-aware algorithms with different number of subscribers. Let the transmission powers of *BS*, *RS*s and *SS*s be 43 dbm (∼ 20 W), 37 dbm (∼ 5 W) and 30 dbm (1 W). Let the traffic ratio of all flows be 1 uniformly. When the number of subscribers increases from 6 to 30, their end-to-end throughput are shown in Figs. 21 and 22. One can see that both of the coding-aware scheduling algorithms have larger throughput compared with their peers. For both fixed power and dynamic power schedulers, the throughput increase or decrease mainly depends on the channel qualities of randomly generated subscribers. They also affect the throughput gains of network coding. One can see in Fig. 22 that the throughput of coding-oblivious and coding-aware schedulers do not increase synchronously. This is mainly because the computation of needed subcarriers in the routing algorithm is not accurate enough when the total powers are dynamically allocated.

In the second set of experiments, we simulate the throughput performance of the OFDMA relay network with different power levels at the *BS*. Let the transmission powers of *RS*s and *SS*s be 37 dbm (∼ 5 W) and 30 dbm (1 W), respectively. The *BS*'s power increases from 36 dbm to 50 dbm. The OFDMA relay network supports 18 randomly distributed subscribers in the circular area. Each subscriber has an uplink flow and a downlink flow, and the traffic ratio of all flows is 1. Fig. 23 shows the aggregate throughput of the coding-oblivious and the coding-aware schemes with fixed power allocation in each subcarrier. Fig. 24 depicts the aggregate throughput of the coding-oblivious and the coding-aware schemes with dynamic power allocation. One can observe that the aggregate throughput increases along with the *BS*'s power for all the scheduling schemes. The coding-aware schedulers outperform the coding-oblivious peers by 4.5–24% in terms of the aggregate throughput. For the fixed-power scheduling, the throughput gain in Fig. 23 increases in the beginning, and shrinks after the BS power is large enough. The simulation results seemingly contradict with the analysis in Lemma 1. This is because the downlink flows do not need relaying when the BS power is very high. The reduced traffic loads due to XOR coding is becoming less and less along with the increase of BS power, resulting in the diminishing throughput gain.

The third set of experiments evaluate the aggregate throughput with different rate ratios. Let the *BS*'s power be 43 dbm (20 W) and the number of subscribers be 18. The remaining settings are the same as those of the second set of experiments. Figs. 25 and 26 illustrate the end-to-end throughput of the proposed algorithms when the traffic ratio of downlink over uplink increases from 0.2 to 3. For the schedulers with fixed power allocation, their end-to-end throughput boost when the downlink/uplink traffic ratio increases. The reason is that the transmission capability of downlink is always greater than the uplink. For the schedulers with dynamic power allocation, the capability of a transmitting node depends on both the node's power and the number of assigned subcarriers. When the downlink traffic load is large, the transmission power of *BS* at a subcarrier is comparatively low, hence limits the end-to-end throughput. As shown in Fig. 26, XOR coding also

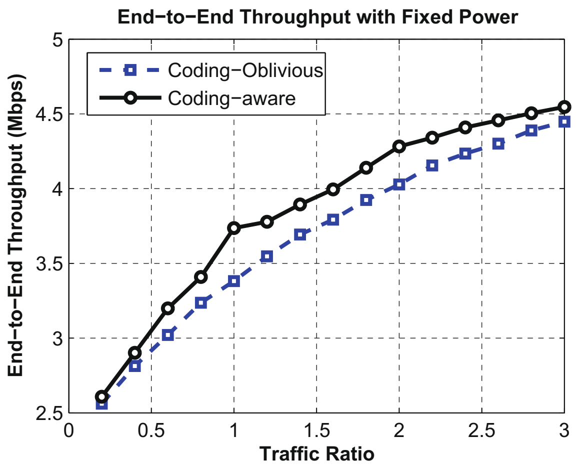

Fig. 25. Traffic ratio VS end-to-end throughput: fixed power allocation.

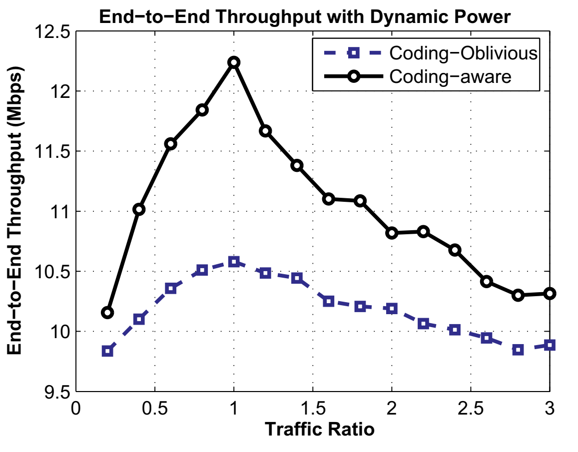

Fig. 26. Traffic ratio VS end-to-end throughput: dynamic power allocation.

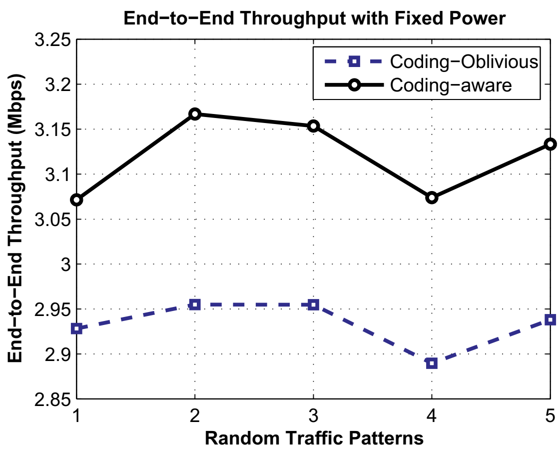

Fig. 27. End-to-end throughput with fixed power allocation.

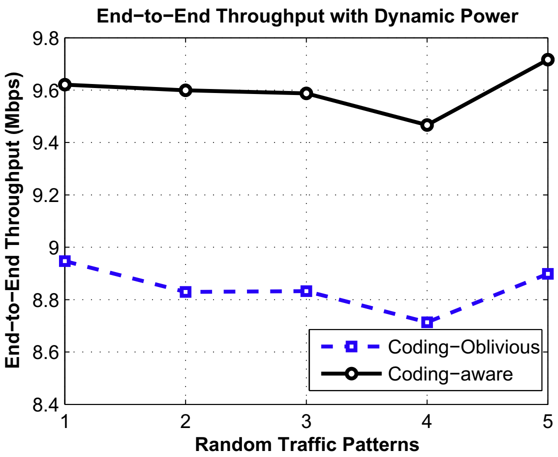

Fig. 28. End-to-end throughput with dyn. power allocation.

improves the end-to-end throughput. Especially, the coding-aware scheduler has a remarkable throughput gain at the point that the downlink/uplink ratio is 1. The throughput gains of XOR coding ranges between 2.8% and 16.3%. We also evaluate the performance of proposed schedulers with five random traffic patterns. The traffic ratios are integer values in the range [0,10]. Fig. 27 plots the end-to-end throughput of the schedulers with fixed power allocation. Fig. 28 plots the end-to-end throughput of the schedulers with dynamic power allocation. Simulation results show that the average throughput gain of XOR coding is 6.4% with the fixed-power scheduling and is 8.7% with the dynamic power scheduling.

7. Related work

Network coding, first proposed in [1], allows interior nodes of a network to not only forward but also process received information. There have been a lot of work to take advantage of the potential benefit of network coding in various settings. Authors in [3,4] investigate the theoretic bounds of throughput gain for single multicast, single/multiple unicast and single broadcast cases in wired network. For the wireless case, [5] provides the upper bounds of throughput gain in 1D and 2D random access networks, and conjectures that the throughput gain is upper bounded by 2 in 2D random networks. A distribute network coding scheme, namely COPE [2], is proposed to bridge theory with practice by using XOR operation locally. The authors also demonstrate the substantial throughput improvement via measurement in the MIT RoofNet. Authors in [6] reveal the relationship between encoding numbers and coding opportunity, and derive the upper bounds of XOR-coding gain under perfect scheduling and random access scenarios. Authors in [8,7] present distributed coding-aware routing protocols to improve throughput performance of wireless ad hoc networks.

However, previous work of wireless network coding are mainly focused either on the 802.11 random access networks or the scheduled Time Division Multiplexing (TDM) networks. Network coding can also be applied to improve the throughput performance of Frequency Division Multiplexing (FDM) systems such as WiMax/802.16j and 3G LTE, which is rarely studied. In [11], authors propose a polynomial time heuristic algorithm to assign subcarriers that optimizes the *maxmin* throughput of bidirectional flows encoded by the base station in a cell. Our work analytically examines the impacts of physical layer parameters on the throughput gain in OFDMA relay structure. Based on the proportional rate ratio scheduling [10,9] in an OFDMA downlink, we propose coding-aware routing, coding-aware channel-aware scheduling algorithms to optimize the capacity for a given traffic pattern in a general OFDMA relay network.

8. Conclusion

Network coding is an important technique to improve wireless network capacity. A practical approach is to XOR packets from bidirectional flows in the relay stations. In this paper, we investigate the routing and scheduling problems in OFDMA relay networks considering the practical network coding scheme. To facilitate XOR operation, we present coding-aware routing metrics to find relay paths with both fixed and dynamic power allocations. Optimization models are formulated to perform resource allocation for an arbitrary traffic pattern. We propose "polynomial time" algorithms to assign subcarriers that exploit not only the benefit of network coding, but also that of multiuser diversity. Our study analytically shows that the throughput gain depends on the transmission power, the channel gains as well as the traffic ratio. Especially, the OFDMA relay network with dynamic power allocation have both **coding gain** and **power gain**. In the future study, we will develop even simpler routing algorithms and partially distributed subcarrier assignment algorithms.

Appendix A. Proof of lemma 1

Proof

(i) If the channel gains satisfy $g_{br} = g_{rb} \geqslant g_{rs} = g_{sr}$, $c_{rb} \geqslant c_{rs}$. Then the network coding gain in Eq. (23) can be rewritten as

$$\frac{\theta_{ca}}{\theta_{co}} = 1 + \frac{1/c_{rb}}{\gamma/c_{br} + 1/c_{sr} + 1/c_{rs} + (\gamma - 1)/c_{rs}} = 1 + \frac{1/c_{rb}}{\gamma/c_{br} + 1/c_{sr} + \gamma/c_{rs}}, \quad (43)$$

where $\gamma = \frac{\gamma_{br}}{\gamma_{sr}}$.
One can easily see that the coding gain is a decreasing function of the traffic ratio γ and the optimal coding gain is achieved at $\gamma = 1$. Similarly, if the channel gain $g_{sr} \geqslant g_{br}$, we have

$$\frac{\theta_{ca}}{\theta_{co}} = 1 + \frac{1/c_{rs}}{\gamma/c_{br} + 1/c_{sr} + 1/c_{rb} + (\gamma - 1)/c_{rs}} = 1 + \frac{1/c_{rb}}{\gamma(1/c_{br} + 1/c_{rs}) + 1/c_{rb} - 1/c_{rs}}. \quad (44)$$

Thus, one can also prove that the optimal traffic ratio for XOR coding is 1.

(ii) We also assume that channel gains satisfy $g_{br} = g_{rb} \geqslant g_{rs} = g_{sr}, c_{rb} \geqslant c_{rs}$. The coding gain can be represented by

$$\frac{\theta_{ca}}{\theta_{co}} = 1 + \frac{1/\log_2\left(1+\frac{P_r g_{rb}}{K}\right)}{\gamma/\log_2\left(1+\frac{P_b g_{br}}{K}\right) + 1/\log_2\left(1+\frac{P_s g_{sr}}{K}\right) + \gamma/\log_2\left(1+\frac{P_r g_{rs}}{K}\right)}. \tag{45}$$

When either P_b or P_s increases, the XOR-coding gain also increase because the denominator decreases.

(iii) Let $g_{rb} = g_{rs} = g_{br} = g_{sr}$ and denote a constant C to be $1/c_{br} + 1/(\gamma c_{sr})$, we have

$$\frac{\theta_{ca}}{\theta_{co}} = 1 + \frac{1}{\gamma} \cdot \frac{1}{C \cdot \log_2(1 + P_r g_{rs}/K) + 1}. \tag{46}$$

When the relay power P_r decreases, the coding gain increases on the contrary. □

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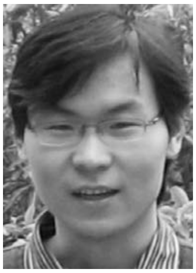

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