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# Pricing Social Visibility Service in Online Social Networks: Modeling and Algorithms

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***Abstract*—In online social networks (OSNs), users may want to enhance their social visibility, as it can make their contents, i.e., opinions, videos, pictures, etc., attract attention from more users. Motivated by this, we propose a mechanism, where the OSN operator provides a *"social visibility boosting service"* to incentivize "transactions" between requesters (users who seek to enhance their social visibility via adding new "neighbors") and suppliers (users who are willing to be added as a new "neighbor" of any requester when certain "rewards" is provided). We design a posted pricing scheme for the OSN provider to charge the requesters who use such boosting service, and reward the suppliers who contribute to such boosting service. The OSN operator keeps a fraction of the payment from requesters and distributes the remaining part to participating suppliers *"fairly"* via a scheme based on the Shapley value. The objective of the OSN provider is to select the price and supplier set to maximize the revenue under the budget constraint of requesters. We first show that the revenue maximization problem is not simpler than an NP-hard problem. We then decompose it into two sub-routines, prove the hardness of each sub-routine, and eventually design computationally efficient approximation algorithms to solve the revenue maximization problem. We conduct extensive experiments to evaluate our proposed algorithms.**

## I. Introduction

OSNs such as YouTube, Instagram, Twitter, Facebook, etc., serve as important platforms for users to share their information or content to friends or followers, e.g., users on Facebook can share their opinions or status to their friends via the friendship network. Users on YouTube can share their videos to their subscribers via the subscriber network. A user's friends or followers can further share the information or content of this user to their friends or followers. Hence, the information or content of a user can be propagated (or "socially visible") to his friends or followers, or even multi-hop friends or followers.

Often times, users want to enhance their social visibility, as it can make their contents, i.e., opinions, videos, pictures, etc., attract attention from more users, which in turn may bring higher commercial benefit to them. We call a user who wants to enhance social visibility as a *requester*. One way to enhance a requester's social visibility is to attract some new friends or followers. It is well known that the OSN is under the "rich gets richer" phenomenon, which makes it difficult for requesters, especially those with low social visibility, to attract new friends or followers. But when finical incentive is provided, some users will be willing to be added as requesters' new friends or followers. We call such users as *suppliers*. Hence, requesters can provide financial incentives to enhance their social visibility. This paper aims to answer the following fundamental question: *How to set appropriate financial incentives to incentivize the "transaction" between requesters and suppliers?*

We propose a mechanism, where the OSN operator provides a *"social visibility boosting service"* to incentivize the transaction between requesters and suppliers. The visibility boosting service is sold via a posted normalized pricing scheme $(p, q)$, where $p \in [0, 1]$ and $q \in [0, 1]$. Here, $p$ is the price of per unit social visibility improvement that the OSN operator charges a participating requester, and $q$ is the price of per unit contribution in improving social visibility of participating requesters that the OSN operator pays to a participating supplier. We consider the case that each participating supplier adds links to all participating requesters and requesters has a budget to add $b \in \mathbb{N}_+$ new friends or followers. Requesters and suppliers decide whether to participate or not by comparing their valuations to the posted prices $(p, q)$. We consider a proportional transaction fee scheme, i.e., the OSN operator keeps a fraction of the payment from requesters, and distribute the remaining part (we call it "reward") to suppliers. The objectives of the OSN operator are: (1) select the price $(p, q)$ and supplier set so to maximize the total amount of transaction fees or equivalently maximize his revenue; (2) divide the reward "fairly" to all suppliers. We address challenges in achieving the above two objectives and our contributions are:

- Formulate a mathematical model to quantify social visibility. To the best of our knowledge, we are the first to propose a posted pricing scheme and formulate a revenue maximization problem for visibility boosting service.
- We decomposed it into two sub-routines, where one focuses on selecting the optimal set of suppliers, and the other focuses on selecting the optimal price. We propose approximation algorithms to solve the problem with provable theoretical guarantee .



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- We show how to divide the reward to suppliers fairly via the Shapley value concept.
- We conduct experiments on real-world social network datasets, and the results validate the effectiveness and efficiency of our algorithms.

## II. Model & Problem Formulation

### A. The Social Visibility Model

**Online Social Network.** Consider an OSN which is characterized by an unweighted and directed graph $\mathcal{G} \triangleq (\mathcal{U}, \mathcal{E})$, where $\mathcal{U} \triangleq \{1, \dots, U\}$ denotes a set of $U \in \mathbb{N}_+$ users and $\mathcal{E} \subseteq \mathcal{U} \times \mathcal{U}$ denotes a set of edges between users. Note that a directed edge from user $u \in \mathcal{U}$ to user $v \in \mathcal{U}$ is denoted by $(u, v) \in \mathcal{E}$. For example, this social network model captures the Twitter social network, the Facebook social network (each undirected edge between user $u$ and $v$ is represented by two directed edges $(u, v)$ and $(v, u)$). We focus on the case that there is no self-loop edges, i.e., $(u, u) \notin \mathcal{E}, \forall u \in \mathcal{U}$.

**Social Visibility.** Denote a directed path in graph $\mathcal{G}$ as $\vec{p} \triangleq (u_0 \to u_1 \to \dots \to u_n)$, where $(u_i, u_{i+1}) \in \mathcal{E}, \forall i \in \{0, \dots, n-1\}$, and $u_i \neq u_j, \forall i, j$. Note that $u_i \neq u_j, \forall i, j$ captures that there is no self-loop edges or circles in the path. Denote a set of all directed edges on path $\vec{p}$ as $\mathcal{F}(\vec{p}) \triangleq \{(u_0, u_1), (u_1, u_2), ..., (u_{n-1}, u_n)\}$. Let $L(\vec{p})$ denote the length (or number of hops) of path $\vec{p}$, which can be expressed as $L(\vec{p}) = |\mathcal{F}(\vec{p})|$. Let $\mathcal{P}(u, v; \mathcal{G})$ denote the set of all directed paths (without circles) from user $u$ to user $v$ in $\mathcal{G}$. Let $D(u, v; \mathcal{G})$ denote the distance from user $u$ to user $v$ in graph $\mathcal{G}$. We define $D(u, v; \mathcal{G})$ as the length of the shortest path from $u$ to $v$, i.e.,

$$D(u, v; \mathcal{G}) \triangleq \begin{cases} \min\limits_{\vec{p} \in \mathcal{P}(u,v;\mathcal{G})} L(\vec{p}), & \text{if } \mathcal{P}(u, v; \mathcal{G}) \neq \emptyset \\ +\infty, & \text{if } \mathcal{P}(u, v; \mathcal{G}) = \emptyset \ . \end{cases}$$

Namely, when there is no directed path from $u$ to $v$, the distance from $u$ to $v$ is infinite. The $d$-visible set of user $u$ is defined as the set of all users to whom user $u$ is $d$-visible, formally

$$\mathcal{V}(u, d; \mathcal{G}) \triangleq \{v | v \in \mathcal{U}, D(v, u; \mathcal{G}) \leq d\}.$$

Let $\tau \in \mathbb{N}_+$ denote the social visibility threshold of an OSN. Based on $\tau$, we define the notation of visibility.

**Definition 1.** *The visibility of user $u$ is the cardinality of his $\tau$-visible set.*

For example, Fig. 1 shows that, user 4's 2-visible set is $\{3, 5, 6\}$ and 3-visible set is $\{3, 5, 6, 7\}$. With a social visibility threshold of $\tau = 2$, user 4's visibility is 3.

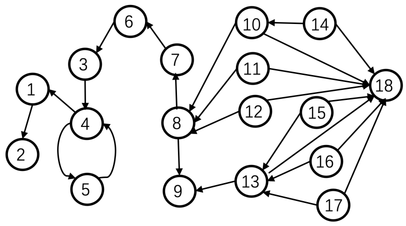



Fig. 1. An example of online social network.

### B. Pricing The Social Visibility

**The pricing scheme.** A *"requester"* is a user in the set $\mathcal{U}$ who seeks to increase his visibility by requesting other users to be his new incoming neighbors. Let $\mathcal{R} \subseteq \mathcal{U}$ denote a set of all requesters. A *"supplier"* is a user in the set $\mathcal{U}$ who is willing to be a new incoming neighbor of any requester. Let $\mathcal{S} \subseteq \mathcal{U}$ denote a set of all suppliers. For the ease of presentation, we assume that $\mathcal{R} \cap \mathcal{S} = \emptyset$, this captures that a user can not be both requester and supplier. We consider the general case that there are some users who are neither requesters nor suppliers, i.e., $\mathcal{R} \cup \mathcal{S} \subseteq \mathcal{U}$. The OSN operator provides a *"social visibility boosting service"* to incentivize the "transaction" between requesters and suppliers. The visibility improvement service is sold at via a posted pricing scheme $(p, q)$, where $p \in [0, 1]$ and $q \in [0, 1]$. Here, $p$ is the price of per unit social visibility improvement that the OSN operator charges a participating requester (i.e., requesters who use this social visibility boosting service), and $q$ is the price of per unit contribution in improving social visibility of participating requesters that the OSN operator pays to a participating supplier. We consider the case that each participating supplier will add links to all participating requesters.

**Requesters' decision model.** Each requester $u \in \mathcal{R}$ has a per unit valuation (i.e., the per unit price that requester $u$ is willing to pay) of $p_u \in [0, 1]$ on the improvement of his social visibility. A requester will use the social visibility boosting service if his per unit valuation is not below the per unit price, i.e., $p_u \geq p$. Let $\widetilde{R}(p)$ denote a set of all participating requesters under price $p$, formally

$$\widetilde{R}(p) \triangleq \{u | p_u \geq p, u \in \mathcal{R}\}.$$

Due to financial budget on boosting service, each requester can afford to add at most $b \in \mathbb{N}_+$ incoming neighbors.

**Supplier's decision model.** Each supplier $u \in \mathcal{S}$ has a per unit valuation (i.e., the per unit price that supplier $u$ is willing to participate) of $q_u \in [0, 1]$ on the per unit contribution in improving social visibility of participating requesters. Let $\mathcal{M}$ denote a set participating suppliers, where $|\mathcal{M}| \leq b$. Let $\widetilde{G}(p, \mathcal{M})$ denote the new graph after adding edges from participating suppliers to participating requesters. $\widetilde{G}(p, \mathcal{M})$ can be expressed as: $\widetilde{G}(p, \mathcal{M}) = (\mathcal{U}, \mathcal{E} \cup (\widetilde{R}(p) \times \mathcal{M}))$. Let $I_u(p, \mathcal{M})$ denote the improvement of social visibility of requester $u \in \widetilde{R}(p)$. It can be expressed as:

$$I_u(p, \mathcal{M}) \triangleq \left| \mathcal{V}(u, \tau; \widetilde{G}(p, \mathcal{M})) \setminus \mathcal{V}(u, \tau; \mathcal{G}) \right| .$$

Then, the total improvement of social visibility of all participating requesters is $I(p,\mathcal{M}) \triangleq \sum_{u\in\widetilde{R}(p)} I_u(p,\mathcal{M})$. Quantifying the contribution to $I(p,\mathcal{M})$ among participating suppliers is a non-trivial problem. There are two underlying challenges: (1) some participating suppliers may have a larger number of follower while others may have a small number of followers; (2) the network structure poses an externality effect, causing the contribution of participating suppliers being correlated. To incentivize suppliers to participate, one needs to divide $I(p,\mathcal{M})$ fairly among suppliers. Let $\phi$ denote a *"fair"* division mechanism, which prescribes a contribution denoted by $\phi_u(p,\mathcal{M})$ for each participating supplier. In order to avoid distracting readers, we defer the detail explanation of $\phi$ to the next section. Given the fair division mechanism $\phi$, a supplier is willing to participant in the social visibility boosting service if his per unit valuation does not exceed the per unit price that the OSN operator pays, i.e., $q_u \le q$. Let $\widetilde{S}(p)$ denote a set of all potential participating suppliers under price $q$, formally

$$\widetilde{S}(q) \triangleq \{u|q_u \le q, u \in \mathcal{S}\}.$$

**Optimal pricing to maximize revenue.** Under the posted pricing scheme, the revenue of the OSN operator is:

$$R(p,q,\mathcal{M}) \triangleq pI(p,\mathcal{M}) - qI(p,\mathcal{M}).$$

We consider a proportional transaction fee scheme, i.e., the OSN operator keeps a fraction $(1-\alpha)$ of requesters' payment $pI(p,\mathcal{M})$, where $\alpha \in (0,1)$ and is fixed. This is equivalent to imposing $q = \alpha p$. We next formulate a revenue maximization problem to select $p, q$ and $\mathcal{M}$.

**Problem 1** (**Optimal pricing to maximize revenue).** *Select $p, q$ and $\mathcal{M}$ to maximize the revenue of the OSN operator:*

$$\begin{aligned} \max_{p,q,\mathcal{M}} \quad & R(p,q,\mathcal{M}) \\ s.t. \quad & \mathcal{M} \subseteq \widetilde{S}(q), |\mathcal{M}| \le b, \\ & q = \alpha p, p \in [0,1], q \in [0,1]. \end{aligned}$$

## III. Algorithms For Optimal Pricing

*Due to page limit, all technical proofs to lemmas and theorems are presented in our supplementary file [1].*

### A. Hardness Analysis

**Hardness analysis.** To illustrate the hardness of Problem 1, we consider a sub-problem of Problem 1 with given pricing parameters $(p,q)$, which is stated as follows:

**Problem 2** (**Optimal supplier set $\mathcal{M}$).** *Given $p$ and $q$ such that $q = \alpha p$ with $0 \le \alpha \le 1$, select $\mathcal{M}$ to maximize the revenue of the OSN operator:*

$$\begin{aligned} \max_{\mathcal{M}} \quad & R(p,q,\mathcal{M}) \\ s.t. \quad & \mathcal{M} \subseteq \widetilde{S}(q), |\mathcal{M}| \le b. \end{aligned}$$

**Theorem 1.** *Problem 2 is NP-hard to solve.*

Note that Problem 2 is a sub-problem of Problem 1. Namely, Problem 1 is harder than Problem 2.

**Our approach.** To address Problem 1, we decompose it into two sub-problems, which each sub-problem serves as a subroutine. In particular, Problem 2 is the first sub-problem. As Theorem 1 shows that Problem 2 is NP-hard, we aim to design an approximation algorithm which we denote as `OptSupplierSet`$(p,q)$ (its detail is postponed to Section III-B). Algorithm `OptSupplierSet`$(p,q)$ takes the prices $(p,q)$ as an input, and returns an approximately optimal set of suppliers under $(p,q)$. We use `OptSupplierSet`$(p,q)$ as an oracle to search for the optimal pricing scheme $(p,q)$. Formally, we aim to solve the following sub-problem:

**Problem 3** (**Optimal price $p,q$).** *Given the algorithm `OptSupplierSet`$(p,q)$, select $(p,q)$ so to maximize the revenue of the OSN operator:*

$$\begin{aligned} \max_{p,q} \quad & R(p,q,\texttt{OptSupplierSet}(p,q)) \\ s.t. \quad & q = \alpha p, p \in [0,1], q \in [0,1]. \end{aligned}$$

We aim to design an approximation algorithm for Problem 3, which we denote as `OptPrice(OptSupplierSet)` (its detail is postponed to Section III-C). One needs to supply `OptPrice` with algorithm `OptSupplierSet`, and `OptPrice(OptSupplierSet)` returns an approximately optimal $(p,q)$. We next proceed to present the design and analysis of `OptSupplierSet`$(p,q)$ and `OptPrice(OptSupplierSet)`.

### B. Design & Analysis of `OptSupplierSet`$(p,q)$

**Submodular analysis**. First, note that once $p$ and $q$ are given, the set of participating requesters $\widetilde{R}(p)$ and the set of potential participating suppliers $\widetilde{S}(q)$ are also set. Our objective is to select $\mathcal{M} \in \widetilde{S}(q)$ with the constraint $|\mathcal{M}| \le b$, so as to maximize the objective function of Problem 2, i.e., revenue $R(p,q,\mathcal{M})$, where $p$ and $q$ are given and $q = \alpha p$. The following theorem shows the sub-modularity and monotonicity of $R(p,q,\mathcal{M})$ with respect to $\mathcal{M}$.

**Theorem 2.** *Given $p$ and $q$, such that $q = \alpha p$, the revenue $R(p,q,\mathcal{M})$ is monotonously increasing and submodular with respect to $\mathcal{M}$.*

**The `OptSupplierSet`$(p,q)$ algorithm.** Based on Theorem 2, Algorithm 1 specifies a greedy algorithm to implement `OptSupplierSet`$(p,q)$. The core idea of Algorithm 1 is that we select suppliers one by one. Each time we select the supplier that achieves the largest marginal improvement in the revenue.

**Algorithm 1** `OptSupplierSet`$(p,q)$

init $\mathcal{M} = \emptyset$
**for** $t = 1$ to $b$ **do**
  $u^* \leftarrow \arg\max_{u\in\widetilde{S}(q)} R(p,q,\mathcal{M}\cup\{u\}) - R(p,q,\mathcal{M})$
  $\mathcal{M} \leftarrow \mathcal{M} \cup \{u^*\}$
**end for**
**return** $\hat{\mathcal{M}}^* \leftarrow \mathcal{M}$

The following theorem presents the theoretical guarantees for Algorithm 1.

**Theorem 3.** *Given $p$ and $q$, such that $q = \alpha p$. The output $\hat{\mathcal{M}}^*$ of Algorithm 1 satisfies: We have*

$$R(p, q, \hat{\mathcal{M}}^*) \geq \left(1 - \frac{1}{e}\right) R(p, q, \mathcal{M}^*),$$

*where $\mathcal{M}^*$ denotes the optimal set of suppliers via exhaustive search.*

### C. Design & Analysis of `OptPrice`(`OptSupplierSet`)

**Discretized search.** The optimal pricing scheme of Problem 3 is not easy to obtain. One challenge is that the closed-form expression of the objective function of Problem 3 w.r.t. $p$ and $q$ is not available. Thus, we resort to the discretized search method. Note that $q = \alpha p$. We therefore discretize the domain of $p$, i.e., $[0, 1]$, uniformly: $\mathcal{A}(\epsilon) \triangleq \{0, \epsilon, 2\epsilon, \dots, \lfloor 1/\epsilon \rfloor \epsilon, 1\}$, where $\epsilon \in (0, 1]$ is price search step. The OSN operator can control $\epsilon$ to adjust the number of points in $\mathcal{A}(\epsilon)$. The OSN operator can use discretized search method to select the optimal price in $\mathcal{A}(\epsilon)$, denoted by $p^*_{\text{DS}}$. Algorithm 2 outlines this discretized search algorithm. We denote $q^*_{\text{DS}} = \alpha p^*_{\text{DS}}$. The set of supplier is then $\hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})$.

**Algorithm 2** `OptPrice(OptSupplierSet)`

Rev $\leftarrow 0$
**for** $p \in \mathcal{A}(\epsilon)$ **do**
  $q \leftarrow \alpha p$
  $\mathcal{M} \leftarrow$`OptSupplierSet`$(p, q)$
  **if** $R(p, q, \mathcal{M}) \geq$ Rev **then**
    $p^*_{\text{DS}} \leftarrow p, q^*_{\text{DS}} \leftarrow q, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}) \leftarrow \mathcal{M}$
    Rev $\leftarrow R(p, q, \mathcal{M})$
  **end if**
**end for**
**return** $p^*_{\text{DS}}, q^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})$

## IV. Algorithms for Fair Division of Contribution

Our results so far can locate the approximate optimal price and supplier set, i.e., $(p^*_{\text{DS}}, q^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$. The remaining issue is how to divide the contribution among participating suppliers fairly. As we mentioned in Section II that a *"fair"* division mechanism is important to incentivize the participation of suppliers. Given the approximate optimal solution $(p^*_{\text{DS}}, q^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$, the OSN operator needs to divide a total contribution of social visibility improvement $I(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$ among all participating suppliers. Note that the naive equal division, i.e., participating suppliers equally share $I(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$ is *not* a fair division. This is because: (1) some participating suppliers may have a larger number of follower while others may have a small number of followers; (2) the network structure poses an externality effect, causing the contribution of participating suppliers being correlated. To achieve fair division, we apply the Shapley value [2] to divide $I(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$. Formally, each participating supplier $u \in \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})$ gets the following share of profit:

$$\begin{aligned}&\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))\\ &= \sum_{\mathcal{M} \subseteq \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}) \setminus \{u\}} \frac{|\mathcal{M}|!(|\hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})| - |\mathcal{M}| - 1)!}{|\hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))|!}\\ &\quad \times (I(p^*_{\text{DS}}, \mathcal{M} \cup \{u\}) - I(p^*_{\text{DS}}, \mathcal{M})).\end{aligned}$$

Readers can refer to [2] for why Shapley value achieves fair division.

One challenge is that the computational complexity of evaluating $\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$ is exponential in the cardinality of $\hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})$. To address this computational challenge, we propose to use the sampling algorithm [3] to approximate $\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$. Let $\sigma = (u_1, \cdots, u_{\widetilde{b}})$ denote an ordering of the participating suppliers, where $\widetilde{b} = \min\{b, |\hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})|\}$ and $u_i \in \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}})$ denotes the participating supplier in the $i$-th order. Denote the set of players ranked before player $i$ in the order $\sigma$ as

$$\mathcal{S}^\sigma_i \triangleq \{\text{all players ranked before } u_i \text{ in the order } \sigma\}.$$

Based on [3], the $\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$ can be rewritten as

$$\begin{aligned}&\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))\\ &= \mathbb{E}_{\sigma \sim \text{Uniform}(\Omega)}[I(p^*_{\text{DS}}, \mathcal{S}^\sigma_i \cup \{u\}) - I(p^*_{\text{DS}}, \mathcal{S}^\sigma_i)], \quad (1)\end{aligned}$$

where $\Omega$ denotes a set of all participating suppliers, and Uniform$(\Omega)$ denotes a uniform distribution over $\Omega$. Based on Equation 1, Algorithm 3 outlines a sampling algorithm to approximate $\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$.

**Algorithm 3** Approximating $\phi_u(p^*_{\text{DS}}, \hat{\mathcal{M}}^*(p^*_{\text{DS}}, q^*_{\text{DS}}))$

$\hat{\phi}_u = 0$
**for** $k = 1$ to $K$ **do**
  generate a ordering $\sigma$ uniformly at random from $\Omega$
  $\hat{\phi}_u \leftarrow [(k-1)\hat{\phi}_u + I(p^*_{\text{DS}}, \mathcal{S}^\sigma_i \cup \{u\}) - I(p^*_{\text{DS}}, \mathcal{S}^\sigma_i)]/k$
**end for**
**return** $\hat{\phi}_u$

**Theorem 4.** *The output $\hat{\phi}_{\sigma_n}$ of Algorithm 3 satisfies*

$$\begin{aligned}&|\hat{\phi}_u - \phi_u(p^*_{DS}, \hat{\mathcal{M}}^*(p^*_{DS}, q^*_{DS}))|\\ &\leq \frac{\max_{\sigma \in \Omega}[I(p^*_{DS}, \mathcal{S}^\sigma_i \cup \{u\}) - I(p^*_{DS}, \mathcal{S}^\sigma_i)]}{\sqrt{K}} \sqrt{\frac{1}{2} \ln \frac{2}{\delta}},\end{aligned}$$

*with a probability of at least $1 - \delta$, where $\delta \in (0, 1]$.*

## V. Performance Evaluation

### A. Experimental Settings

**Datasets.** We evaluate our algorithms on two public datasets, i.e., Blogs [4] and DBLP [5], whose overall statistics is summarized in Table I.

**Parameter setting.** To reflect the real-world setting that only a small portion of users in an OSN is interested in the social visibility boosting service, we select $\gamma$ fraction of users

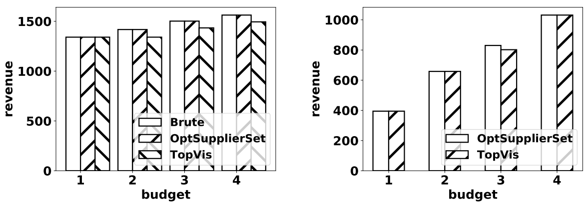
TABLE I
STATISTICS OF FOUR DATASETS.

| datasets | #nodes | #links | type | $\tau$ | $\gamma$ |
|---|---|---|---|---|---|
| Blogs | 1,224 | 19,025 | directed | 2 | 0.5 |
| DBLP | 10,000 | 55,734 | undirected | 2 | 0.05 |

uniformly at random from the user population $\mathcal{U}$ as requesters $\mathcal{R}$, and another $\gamma$ fraction as suppliers $\mathcal{S}$, where $\gamma \in [0, 0.5]$. Valuation $p_u$ is sampled from Beta distribution $B(3, 6)$ for each requester $u \in \mathcal{R}$, and valuation $q_u$ is sampled from Beta distribution $B(6, 3)$ for each supplier $u \in \mathcal{S}$. Throughout the experiments, we fixed $\alpha = 0.6$, i.e., the OSN operator keeps 40% of the payment from the requesters as the transaction fee.

### B. Evaluating `OptSupplierSet`

Under each given pricing scheme $(p, q)$, we compare our `OptSupplierSet` algorithm, i.e., Algorithm 1, with the following two baselines: (1) `Brute` which selects the optimal set of participating suppliers via exhaustive search; and (2) `TopVis` which selects suppliers with the top-$b$ social visibility from the potential supplier set. Note that algorithm `Brute` is computationally expensive, we only experiment it on small dataset Blogs. Figure 2 shows the revenue achieved by different methods of finding optimal supplier set, where we fix search step as $\epsilon = 0.025$. From Figure 2, one can observe that the revenue under our `OptSupplierSet` algorithm is nearly the same as that under the `Brute` algorithm on dataset Blogs, and outperforms heuristic algorithm `TopVis` on both datasets. This implies a high accuracy of our `OptSupplierSet` in approximating the optimal supplier set.




(a) Blogs (b) DBLP

Fig. 2. Revenue under different methods to find optimal supplier set.

## VI. Related Work

The notion of social visibility defined in this paper is closely related to social influence [6]–[9]. One key difference is that when a user is visible to a set of users, it does not mean this user can influence this set of users. The objective of influence maximization problem is to find a subset of nodes that could maximize the spread of information under certain influence diffusion models. But our problem focuses on the pricing of social visibility service. The idea of adding new links to enhance social visibility is closely related to link prediction [10]–[12], and friend recommendation [13]–[16], The objectives of link prediction and friend recommendation are to predict future or missing links. However, our work adds links that can improve social visibility. Note that such links may have nothing to do with predicting the future or missing links.

## VII. Conclusions

This paper proposes a mechanism where the OSN operator prices its social visibility boosting service judiciously to maximize his revenue. We formulate a revenue maximization problem for the OSN operator to select the parameter of the posted pricing scheme. We show that the revenue maximization problem is not simpler than an NP-hard problem. We decomposed it into two sub-routines, where one focuses on selecting the optimal set of suppliers, and the other one focuses on selecting the optimal prices. We prove the hardness of each sub-routine, and eventually design a computationally efficient approximation algorithm to solve the revenue maximization problem. We conduct experiments on public datasets to validate the superior performance of our proposed algorithms.

## VIII. Acknowledgment

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## REFERENCES

[1] S. Zheng, H. Xie, and J. Lui, "Pricing social visibility service in online social networks: Modeling and algorithms," *arXiv preprint arXiv:2106.10473*, 2021.
[2] L. S. Shapley, *Notes on the N-person Game–II: The Value of an N-person Game*. Rand Corporation, 1951.
[3] Y. Bachrach, E. Markakis, E. Resnick, A. D. Procaccia, J. S. Rosenschein, and A. Saberi, "Approximating power indices: theoretical and empirical analysis," *Autonomous Agents and Multi-Agent Systems*, vol. 20, no. 2, pp. 105–122, 2010.
[4] J. Kunegis, "KONECT – The Koblenz Network Collection," in *Proc. Int. Conf. on World Wide Web Companion*, 2013, pp. 1343–1350.
[5] W. Nawaz, K.-U. Khan, Y.-K. Lee, and S. Lee, "Intra graph clustering using collaborative similarity measure," *Distributed and Parallel Databases*, vol. 33, no. 4, pp. 583–603, 2015.
[6] D. Kempe, J. Kleinberg, and É. Tardos, "Maximizing the spread of influence through a social network," in *Proc. ACM KDD*, 2003.
[7] P. Domingos and M. Richardson, "Mining the network value of customers," in *Proc. ACM KDD*. ACM, 2001.
[8] Y. Lin, W. Chen, and J. C. Lui, "Boosting information spread: An algorithmic approach," in *2017 IEEE 33rd International Conference on Data Engineering (ICDE)*. IEEE, 2017, pp. 883–894.
[9] M. Richardson and P. Domingos, "Mining knowledge-sharing sites for viral marketing," in *Proc. ACM KDD*. ACM, 2002.
[10] L. L and T. Zhou, "Link prediction in complex networks: A survey," *Physica A: Statistical Mechanics and its Applications*, vol. 390, no. 6, p. 11501170, Mar 2011.
[11] V. Martínez, F. Berzal, and J.-C. Cubero, "A survey of link prediction in complex networks," *ACM CSUR*, vol. 49, no. 4, p. 69, 2017.
[12] D. Liben-Nowell and J. Kleinberg, "The link-prediction problem for social networks," *Journal of the American society for information science and technology*, vol. 58, no. 7, pp. 1019–1031, 2007.
[13] P. Resnick and H. R. Varian, "Recommender systems," *Communications of the ACM*, vol. 40, no. 3, pp. 56–59, 1997.
[14] H. Ma, "On measuring social friend interest similarities in recommender systems," in *Proc. ACM SIGIR*. ACM, 2014.
[15] H. Ma, D. Zhou, C. Liu, M. R. Lyu, and I. King, "Recommender systems with social regularization," in *Proc. WSDM*. ACM, 2011.
[16] X. Xie, "Potential friend recommendation in online social network," in *Proc. IEEE/ACM CGCC*. IEEE Computer Society, 2010.