Table I.1
Some Properties of the z-Transform

SEQUENCE		z-TRANSFORM
1. $f_n \quad n = 0, 1, 2, \ldots$	$\Leftrightarrow$	$F(z) = \sum_{n=0}^{\infty} f_n z^n$
2. $af_n + bg_n$		$aF(z) + bG(z)$
3. $a^n f_n$		$F(az)$
4. $f_{n/k} \quad n = 0, k, 2k, \ldots$		$F(z^k)$
5. f_{n+1}		$\frac{1}{z}[F(z) - f_0]$
6. $f_{n+k} \quad k > 0$		$\frac{F(z)}{z^k} - \sum_{i=1}^{k} z^{i-k-1} f_{i-1}$
7. f_{n-1}		$zF(z)$
8. $f_{n-k} \quad k > 0$		$z^k F(z)$
9. nf_n		$z \frac{d}{dz} F(z)$
10. $n(n-1)(n-2), \ldots, (n-m+1)f_n$		$z^m \frac{d^m}{dz^m} F(z)$
11. $f_n \circledast g_n$		$F(z)G(z)$
12. $f_n - f_{n-1}$		$(1 - z)F(z)$
13. $\sum_{k=0}^{n} f_k \quad n = 0, 1, 2, \ldots$		$\frac{F(z)}{1 - z}$
14. $\frac{\partial}{\partial a} f_n$ (*a* is a parameter of f_n)		$\frac{\partial}{\partial a} F(z)$
15. Series sum property		$F(1) = \sum_{n=0}^{\infty} f_n$
16. Alternating sum property		$F(-1) = \sum_{n=0}^{\infty} (-1)^n f_n$
17. Initial value theorem		$F(0) = f_0$
18. Intermediate value theorem		$\frac{1}{n!} \frac{d^n F(z)}{dz^n} \Big\vert_{z=0} = f_n$
19. Final value theorem		$\lim_{z \to 1} (1 - z)F(z) = f_\infty$

Table I.2
Some z-Transform Pairs

SEQUENCE		z-TRANSFORM
1. $f_n \quad n = 0, 1, 2, \ldots$	$\Leftrightarrow$	$F(z) = \sum_{n=0}^{\infty} f_n z^n$
2. $u_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$		1
3. u_{n-k}		z^k
4. $\delta_n = 1 \quad n = 0, 1, 2, \ldots$		$\dfrac{1}{1 - z}$
5. δ_{n-k}		$\dfrac{z^k}{1 - z}$
6. $A\alpha^n$		$\dfrac{A}{1 - \alpha z}$
7. $n\alpha^n$		$\dfrac{\alpha z}{(1 - \alpha z)^2}$
8. n		$\dfrac{z}{(1 - z)^2}$
9. $n^2\alpha^n$		$\dfrac{\alpha z(1 + \alpha z)}{(1 - \alpha z)^3}$
10. n^2		$\dfrac{z(1 + z)}{(1 - z)^3}$
11. $(n + 1)\alpha^n$		$\dfrac{1}{(1 - \alpha z)^2}$
12. $(n + 1)$		$\dfrac{1}{(1 - z)^2}$
13. $\dfrac{1}{m!}(n + m)(n + m - 1) \cdots (n + 1)\alpha^n$		$\dfrac{1}{(1 - \alpha z)^{m+1}}$
14. $\dfrac{1}{n!}$		e^z

In words, then, we have that the z-transform of the convolution of two sequences is equal to the product of the z-transform of each of the sequences themselves.

In Table I.1 we list a number of important properties of the z-transform, and following that in Table I.2 we provide a list of important common z-transforms.

Table I.3
Some Properties of the Laplace Transform

FUNCTION		TRANSFORM
1. $f(t) \quad t \geq 0$	$\Leftrightarrow$	$F^*(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$
2. $af(t) + bg(t)$		$aF^*(s) + bG^*(s)$
3. $f\left(\frac{t}{a}\right) \quad (a > 0)$		$aF^*(as)$
4. $f(t-a)$		$e^{-as}F^*(s)$
5. $e^{-at}f(t)$		$F^*(s+a)$
6. $tf(t)$		$-\frac{dF^*(s)}{ds}$
7. $t^n f(t)$		$(-1)^n \frac{d^n F^*(s)}{ds^n}$
8. $\frac{f(t)}{t}$		$\int_{s_1=s}^{\infty} F^*(s_1)\,ds_1$
9. $\frac{f(t)}{t^n}$		$\int_{s_1=s}^{\infty} ds_1 \int_{s_2=s_1}^{\infty} ds_2 \cdots \int_{s_n=s_{n-1}}^{\infty} ds_n F^*(s_n)$
10. $f(t) \circledast g(t)$		$F^*(s)G^*(s)$
11.† $\frac{df(t)}{dt}$		$sF^*(s)$
12.† $\frac{d^n f(t)}{dt^n}$		$s^n F^*(s)$
13.† $\int_{-\infty}^{t} f(t)\,dt$		$\frac{F^*(s)}{s}$
14.† $\underbrace{\int_{-\infty}^{t} \cdots \int_{-\infty}^{t}}_{n \text{ times}} f(t)(dt)^n$		$\frac{F^*(s)}{s^n}$
15. $\frac{\partial}{\partial a} f(t)$ [a is a parameter]		$\frac{\partial}{\partial a} F(s)$
16. Integral property		$F^*(0) = \int_{0-}^{\infty} f(t)\,dt$
17. Initial value theorem		$\lim_{s\to\infty} sF^*(s) = \lim_{t\to 0} f(t)$
18. Final value theorem		$\lim_{s\to 0} sF^*(s) = \lim_{t\to\infty} f(t)$ if $sF^*(s)$ is analytic for $\text{Re}(s) \geq 0$

† To be complete, we wish to show the form of the transform for entries 11–14 in the case when $f(t)$ may have nonzero values for $t < 0$ also:

$$\frac{d^n f(t)}{dt^n} \Leftrightarrow s^n F^*(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-) - \cdots - f^{(n-1)}(0^-)$$

$$\underbrace{\int_{-\infty}^{t} \cdots \int_{-\infty}^{t}}_{n \text{ times}} f(t)(dt)^n \Leftrightarrow \frac{F^*(s)}{s^n} + \frac{f^{(-1)}(0^-)}{s^n} + \frac{f^{(-2)}(0^-)}{s^{n-1}} + \cdots + \frac{f^{(-n)}(0^-)}{s}$$

Table I.4
Some Laplace Transform Pairs

FUNCTION		TRANSFORM
1. $f(t) \quad t \geq 0$	$\Leftrightarrow$	$F^*(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$
2. $u_0(t)$ (unit impulse)		1
3. $u_0(t-a)$		e^{-as}
4. $u_n(t) \triangleq \frac{d}{dt} u_{n-1}(t)$		s^n
5. $u_{-1}(t) \triangleq \delta(t)$ (unit step)		$\frac{1}{s}$
6. $u_{-1}(t-a)$		$\frac{e^{-as}}{s}$
7. $u_{-n}(t) \triangleq \frac{t^{n-1}}{(n-1)!}$		$\frac{1}{s^n}$
8. $Ae^{-at}\,\delta(t)$		$\frac{A}{s+a}$
9. $te^{-at}\,\delta(t)$		$\frac{1}{(s+a)^2}$
10. $\frac{t^n}{n!} e^{-at}\,\delta(t)$		$\frac{1}{(s+a)^{n+1}}$

whereas Property 5, its dual, gives the effect of a parameter shift in the transform domain. Properties 6 and 7 show the effect of multiplication by t (to some power), which corresponds to differentiation in the transform domain; similarly, Properties 8 and 9 show the effect of division by t (to some power), which corresponds to integration. Property 10, a most important property (derived earlier), shows the effect of convolution in the time domain going over to simple multiplication in the transform domain. Properties 11 and 12 give the effect of time differentiation; it should be noted that this corresponds to multiplication by s (to a power equal to the number of differentiations in time) times the original transform. In a similar way Properties 13 and 14 show the effect of time integration going over to division by s in the transform domain. Property 15 shows that differentiation with respect to a parameter of $f(t)$ corresponds to differentiation in the transform domain as well. Property 16, the integral property, shows the simple way in which the transform may be evaluated at the origin to give the total integral