CSC5420

TAKE HOME MIDTERM EXAMINATION, 2005-2006

Problem	Points	Score
problem 1	8	
problem 2	8	
problem 3	8	
problem 4	8	
problem 5	8	
problem 6	8	
problem 7	8	
problem 8	8	
problem 9	8	
problem 10	8	
problem 11	10	
problem 12	10	
Total:	100	

Student's name: ______________________________

Student's ID: ________________________________

1. Three coins are tossed: two nickels and a dime. We pay \$0.1 to enter the game and receive those coins which falls heads up. What is the probability of winning some money?

2. Two students have an appointment with their tutor at noon. The first student makes an effort to be on time, but he lives far from the university and a recent rain storm in Hong Kong has paralyzed the city. He may be late but will certainly make it before 1 p.m. The time of his arrival depends on so many unpredictable factors that we regard it as random. The other student lives next to the university, but he forgot about the appointment and may come at any time after having remembering it. Let D_i be the random variable denoting the delay of the student i, $i = 1, 2$. Find the probability density functions $f_{D_i}(x)$ for $i = 1, 2$ for both students.

3. The people are randomly seated at a round table. What is the probability that a particular couple will sit next to each other?

4. A student has to sit for an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass, he needs to answer all three questions. What is the probability that the student will pass the examination if he knows the answers to 90 questions on the list?

5. You have three coins in your pocket, two fair ones but the third biased with probability of heads p and tails $1 - p$. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins?

6. Let X be the number of tosses of a fair coin up to and including the first toss showing heads. Find $P(\{X \in 2N\})$, where $2N = \{2n : n = 1, 2, 3, \ldots\}$ is the set of even integers.

7. Two dice are rolled. Let X be the larger of the two numbers shown. Compute $P_X([2, 4])$.

8. Two towns A and B are 50 miles apart. Car 1 leaves town A and car 2 leaves town B independently of one another. The departure time of each car is random and uniformly distributed between 12 and 1 o'clock. The speed of each car is 100 miles per hour and they travel directly to meet one another. Denote by X the distance between the meeting point from A. Find F_X.

9. Show that $|E(X)| \leq E(|X|)$ for a random variable X.

10. A coin is tossed repeatedly until a tail is obtained and you win a^k dollars, where k is the number of heads before the first tail and $a > 0$. How much would you consider a fair amount to pay to play this game?

11. Let X be a random variable with the Poisson distribution. Find the conditional expectation of X given that X is an even number.

12. Consider a system that is identical to an M/M/1 with arrival rate λ and service rate μ except that when the system empties out service, it does not begin again until k customers are present in the system (where k is a given parameter). Once service begins, it proceeds normally until the system becomes empty again.

(a) Draw the state transition diagram of this queueing system

(b) Find the stead state probability vector $\boldsymbol{\pi}$.

(c) Find the average number of customers in the system.

(d) Find the average response time of customers in the system.